VIGRE: Vertical InteGration of Research and Education in the Mathematical Sciences

- Major NSF initiative to nurture young mathematicians nationwide.
- Original Rice grant from 2003 – 2008; followed-on for 2008 – 2013.
- Provides funding for undergraduate research, graduate students, postdocs.
- Rice project integrates three departments:
  - Computational and Applied Mathematics
  - Mathematics
  - Statistics
- Working groups are organized into PFUGs (fugues):
  - **P**ostdoc
  - **F**aculty
  - **U**ngergraduate
  - **G**raduate
- A diverse set of (often interdisciplinary) PFUGs organize seminars during semesters, summers.

VIGRE: A Sample of PFUGs

- Computational Algebraic Geometry
- Computational Finance (Electronic Market Dynamics)
- Differential Geometry
- Edge-Length Minimizing Polyhedra
- Heegaard Floer Homology for Knots and 3-Manifolds
- Medical Image Processing
- Neuronal Networks of the Hippocampus
- Physics of Strings
- Statistical Bioinformatics
- Stochastic Modeling in Systems Biology

VIGRE: Challenges of Continuity and Documentation

The PFUGs face a number of challenges in documenting and disseminating their research.

- Membership varies from semester to semester, particularly among undergraduates.
- PFUGs must transmit a growing body of knowledge to new members without starting from scratch each semester.
- A centralized web site has not proved effective:
  - access restrictions add a barier to maintenance;
  - burden falls to group leaders;
  - different PFUGs have different styles.
- Now PFUGs typically maintain their own local, publicly-accessible web sites with seminar announcements, working notes, software, etc.
- Some PFUGs use our local OwlSpace collaboration tool (Sakai).

- We would still like some way to publicly archive summaries of each PFUG's work each semester.

Solution: The Art of the PFUG Collection in Connexions

- Connexions provides a great collaborative tool for collecting information from the various PFUGs.
- Students typically draft in $\LaTeX$ (which they know, or learn), then import into CNXML.

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VIGRE: The Art of the PFUG

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- Students typically draft in LaTeX (which they know, or learn), then import into CNXML.
- Spring/Summer 2008: two new research modules
  - Modeling Cell Assemblies
  - The Network Wave Equation
- Fall 2008: six new research modules
  - Experiments with the Three-Spectral Inverse Problem for a Beaded String
  - Edge Length Minimizing Polyhedra
  - Image Denoising via the Redundant Wavelet Transform
  - A Class of Fast Algorithms for Total Variation Image Restoration
  - Michell Trusses
  - Dynamics of the Firing Rate of Single Compartment Cells

  [LaTeX template to help groups write convertible source code.]

VIGRE: Three-Spectral Inverse Problem for a Beaded String

Mechanical Engineering undergrad Hunter Gilbert (Physics of Strings PFUG)

INSIDE COLLECTION:

The Art of the PFUG

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Collection by: Steven Cox

Experiments with the Three-Spectral Inverse Problem for a Beaded String

Module by: Hunter Gilbert

Summary: This report summarizes work done as part of the Physics of Strings PFUG under Rice University's VIGRE program. VIGRE is a program of Vertically Integrated Grants for Research and Education in the Mathematical Sciences under the direction of the National Science Foundation. A PFUG is a group of Postdocs, Faculty, Undergraduates and Graduate students formed round the study of a common problem. This module describes the three-spectral inverse problem for a beaded string and presents experimental results of its application.

Introduction

How well can we predict a string's mass distribution by simply listening to its vibration? While considering this question, previous experiments have been limited in the types of strings that could be studied. When only considering two spectra (fixed-fixed and fixed-flat), acquiring the necessary data required us to force the beaded strings to be symmetric about the midpoint. This condition has severely limited the possible experiments. However, recent theoretical developments by Boyko and Pivovarchik **Entry 61** have expanded the regime of experimental work with beaded strings. Here we consider three fixed-fixed spectra (whole string, clamped left section, and clamped right section), and show that the information contained in these three spectra may be written as two sets of two spectra problems. Thus, for an arbitrary beaded string, it is possible to measure the frequencies of vibration of three sections of the string. It is then possible to convert these spectra into two separate inverse problems with well known solutions. An algorithm for the recovery of the length and mass information of the string is given by Cox, et. al. **Entry 62**. Here is presented the theoretical framework and an experimental setup to predict the masses and lengths of any arbitrary beaded string as long as the string meets our much shorter list of requirements.

VIGRE: Three-Spectral Inverse Problem for a Beaded String

Mechanical Engineering undergrad Hunter Gilbert (Physics of Strings PFUG)

The Three-Spectral Inverse Problem

Denote the spectra of the unclamped string by λ_k, and the spectra of the left and right parts by $\nu_{k,\ell}$ and $\nu_{k,r}$. L is the length of the whole string and L_ℓ and L_r are the lengths of the separate parts. From this information we immediately construct three polynomials:

$$p_w(\lambda^2) = L\prod_{k=1}^{n_1+n_2}\left(1-\frac{\lambda^2}{\lambda_k^2}\right), \quad p_\ell(\lambda^2) = L_\ell\prod_{k=1}^{n_1}\left(1-\frac{\lambda^2}{\nu_{k,\ell}^2}\right), \quad p_r(\lambda^2) = L_r\prod_{k=1}^{n_2}\left(1-\frac{\lambda^2}{\nu_{k,r}^2}\right) \quad (5)$$

Note that $p_w(\lambda^2)$ is proportional to $\phi(\lambda^2)$, $p_\ell(\lambda^2)$ is proportional to $R_{2n_1}(\lambda^2)$, and p_r is proportional to $\tilde{R}_{2n_2}(\lambda^2)$. It is known that the ratio of polynomials $R_{2n_1}(z)/R_{2n_1-1}(z)$ has the continued fraction expansion:

$$\frac{R_{2n_1}(\lambda^2)}{R_{2n_1-1}(\lambda^2)} = \ell_n + \cfrac{1}{-\frac{m_n}{\sigma}\lambda^2 + \cfrac{1}{\ell_{n-1} + \cfrac{1}{-\frac{m_{n-1}}{\sigma}\lambda^2 + \dots + \cfrac{1}{\ell_1 + \cfrac{1}{-\frac{m_1}{\sigma}\lambda^2 + \frac{1}{\ell_0}}}}}} \quad (6)$$

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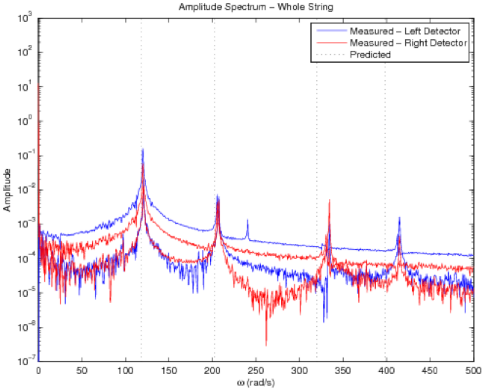
Experimental Results

Experimental testing has given very positive results. Despite the difficulties in collecting real data, the inverse algorithm just described has produced reasonable bead masses and lengths of string between the beads.

The peaks of the FFT plots shown in Figures **Figure 5**, **Figure 6**, and **Figure 7**, give the observed frequencies for each section of the string. These are the eigenvalues that are used to construct the polynomials used during the inversion procedure.

Two sets of lenghts and masses that we recovered are depicted visually in figures **Figure 9** and **Figure 11**.



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Results for a string with 4 beads

Figure 10: Measured

Figure 11: Recovered

	m_1	m_2	$\tilde{m}_1$	$\tilde{m}_2$	ℓ_1	ℓ_2	ℓ_3	$\tilde{\ell}_1$	$\tilde{\ell}_2$	$\tilde{\ell}_3$
Measured	30.8	17.8	30.8	30.8	0.127	0.229	0.092	0.362	0.152	0.162
Recovered	27.7	17.2	29.6	29.0	0.139	0.226	0.082	0.368	0.150	0.159

TABLE 2

How Connexions has helped our VIGRE project

Connexions has given us:

- a more systematic way of archiving our disparate research projects;
- a tool for recruiting students to research groups;
- a simple, unified way to disseminate these projects to the public;
- assistance in documenting, reporting our results to NSF.

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