Signals, Systems, and Transforms (under construction)

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C O N N E X I O N S

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Preface: Signals and Systems[1]

In this course, we will learn about signals and systems for processing signals. We will rely heavily on ideas from linear algebra and Hilbert space to unify our treatment of the four fundamental classes of signals - the four combinations of discrete-time vs. continuous-time and periodic/finite vs. aperiodic/infinite.

In contrast to most textbooks, we begin with the discrete Fourier transform for discrete-time, periodic/finite signals. Hence, this course could be subtitled: **DFT First**. For a more standard treatment starting with continuous-time signals, see the Connexions course Signals and Systems[2] which was used until 2002 and in 2004.

This course also marks the introduction of National Instruments **Labview VIs**[3] to the Connexions system. Look for them in select modules - they are designed to help students visualize important concepts.

Comments, typos, and suggestions are welcome.

[1]This content is available online at <http://cnx.org/content/m11483/1.6/>.
[2]http://cnx.rice.edu/content/col10064/latest/
[3]http://ni.com/labview

Chapter 1

Introduction to Signals, Systems, and Transforms

1.1 Signals[1]

1.2 Operators[2]

1.2.1 Operators

1.2.2 Systems

systems "process"/change

1.2.3 Transforms

"re-express/translate"

1.3 System Classifications and Properties[3]

1.3.1 Introduction

In this module some of the basic classifications of systems will be briefly introduced and the most important properties of these systems are explained. As can be seen, the properties of a system provide an easy way to separate one system from another. Understanding these basic difference's between systems, and their properties, will be a fundamental concept used in all signal and system courses, such as digital signal processing (DSP). Once a set of systems can be identified as sharing particular properties, one no longer has to deal with proving a certain characteristic of a system each time, but it can simply be accepted do the the systems classification. Also remember that this classification presented here is neither exclusive (systems can belong to several different classifications) nor is it unique (there are other methods of classification [4]). Examples of simple systems can be found here[5].

[1] This content is available online at <http://cnx.org/content/m11501/1.2/>.
[2] This content is available online at <http://cnx.org/content/m11499/1.4/>.
[3] This content is available online at <http://cnx.org/content/m10084/2.20/>.
[4] "Introduction to Systems" <http://cnx.org/content/m0005/latest/>
[5] "Simple Systems" <http://cnx.org/content/m0006/latest/>

1.3.2 Classification of Systems

Along with the classification of systems below, it is also important to understand other Classification of Signals[6].

1.3.2.1 Continuous vs. Discrete

This may be the simplest classification to understand as the idea of discrete-time and continuous-time is one of the most fundamental properties to all of signals and system. A system where the input and output signals are continuous is a **continuous system**, and one where the input and output signals are discrete is a **discrete system**.

1.3.2.2 Linear vs. Nonlinear

A **linear** system is any system that obeys the properties of scaling (homogeneity) and superposition (additivity), while a **nonlinear** system is any system that does not obey at least one of these.

To show that a system H obeys the scaling property is to show that

$$H\left(kf\left(t\right)\right)=kH\left(f\left(t\right)\right) \tag{1.1}$$

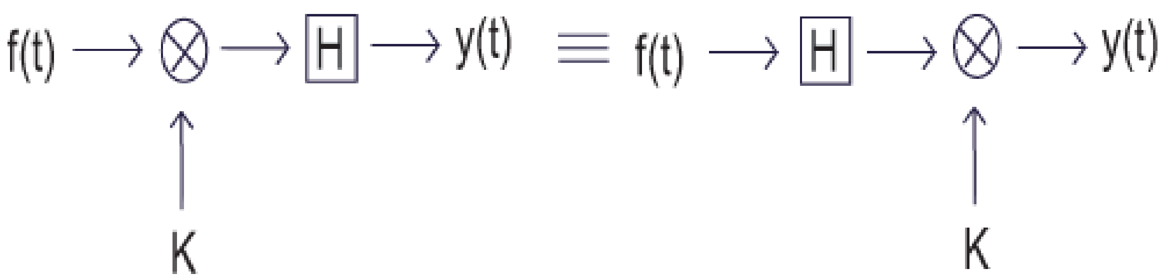


Figure 1.1: A block diagram demonstrating the scaling property of linearity

To demonstrate that a system H obeys the superposition property of linearity is to show that

$$H\left(f_1\left(t\right)+f_2\left(t\right)\right)=H\left(f_1\left(t\right)\right)+H\left(f_2\left(t\right)\right) \tag{1.2}$$

[6]"Signal Classifications and Properties" <http://cnx.org/content/m10057/latest/>

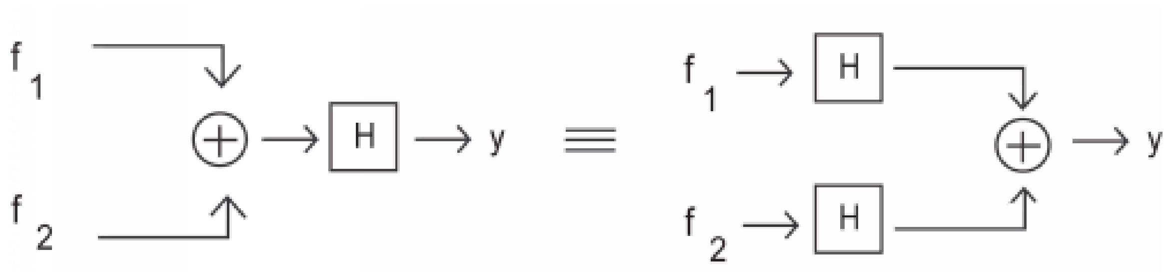


Figure 1.2: A block diagram demonstrating the superposition property of linearity

It is possible to check a system for linearity in a single (though larger) step. To do this, simply combine the first two steps to get

$$H\left(k_1 f_1\left(t\right) + k_2 f_2\left(t\right)\right) = k_2 H\left(f_1\left(t\right)\right) + k_2 H\left(f_2\left(t\right)\right) \tag{1.3}$$

1.3.2.3 Time Invariant vs. Time Variant

A **time invariant** system is one that does not depend on when it occurs: the shape of the output does not change with a delay of the input. That is to say that for a system H where $H\left(f\left(t\right)\right) = y\left(t\right)$, H is time invariant if for all T

$$H\left(f\left(t-T\right)\right) = y\left(t-T\right) \tag{1.4}$$

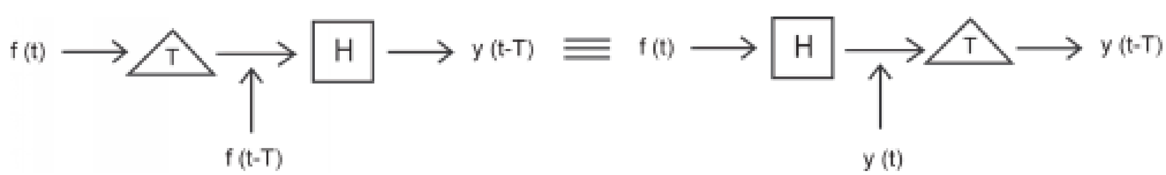


Figure 1.3: This block diagram shows what the condition for time invariance. The output is the same whether the delay is put on the input or the output.

When this property does not hold for a system, then it is said to be **time variant**, or time-varying.

1.3.2.4 Causal vs. Noncausal

A **causal** system is one that is **nonanticipative**; that is, the output may depend on current and past inputs, but not future inputs. All "realtime" systems must be causal, since they can not have future inputs available to them.

One may think the idea of future inputs does not seem to make much physical sense; however, we have only been dealing with time as our dependent variable so far, which is not always the case. Imagine rather that we wanted to do image processing. Then the dependent variable might represent pixels to the left and right (the "future") of the current position on the image, and we would have a **noncausal** system.

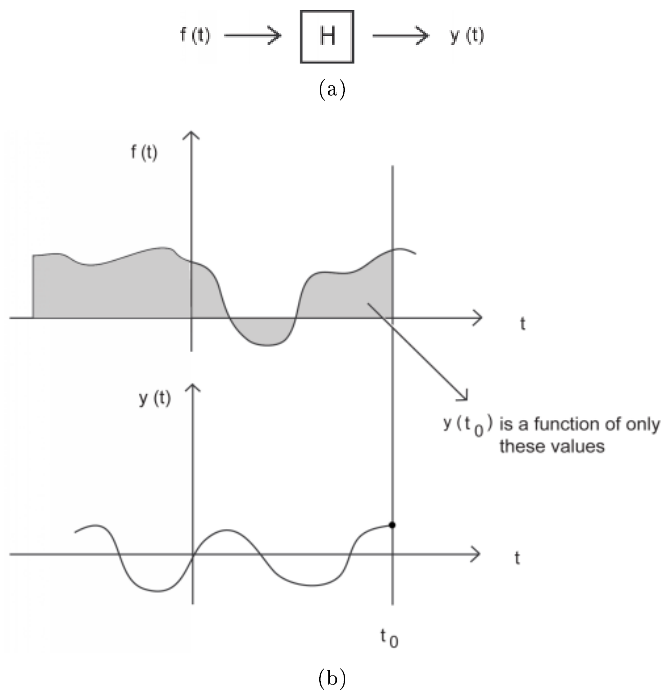


Figure 1.4: (a) For a typical system to be causal... (b) ...the output at time t_0, $y(t_0)$, can only depend on the portion of the input signal before t_0.

1.3.2.5 Stable vs. Unstable

A **stable** system is one where the output does not diverge as long as the input does not diverge. There are many ways to say that a signal "diverges"; for example it could have infinite energy. One particularly useful definition of divergence relates to whether the signal is bounded or not. Then a system is referred to as **bounded input-bounded output (BIBO)** stable if **every possible** bounded input produces a bounded output.

Representing this in a mathematical way, a stable system must have the following property, where $x(t)$ is the input and $y(t)$ is the output. The output must satisfy the condition

$$|y(t)| \leq M_y < \infty \tag{1.5}$$

when we have an input to the system that can be described as

$$|x(t)| \leq M_x < \infty \tag{1.6}$$

M_x and M_y both represent a set of finite positive numbers and these relationships hold for all of t.

If these conditions are not met, *i.e.* a system's output grows without limit (diverges) from a bounded input, then the system is **unstable**. Note that the BIBO stability of a linear time-invariant system (LTI) is neatly described in terms of whether or not its impulse response is absolutely integrable[7].

1.4 Transforms[8]

Add links examples such as Laplace, fourier and wavelets

[7] "BIBO Stability" <http://cnx.org/content/m10113/latest/>
[8] This content is available online at <http://cnx.org/content/m11500/1.2/>.

Chapter 2

Signals

2.1 Signal Basics

2.1.1 Signals are functions[1]

A signal is a function that maps an independent variable into a dependent variable. The function f(x), for each value of x, produces the value f(x)

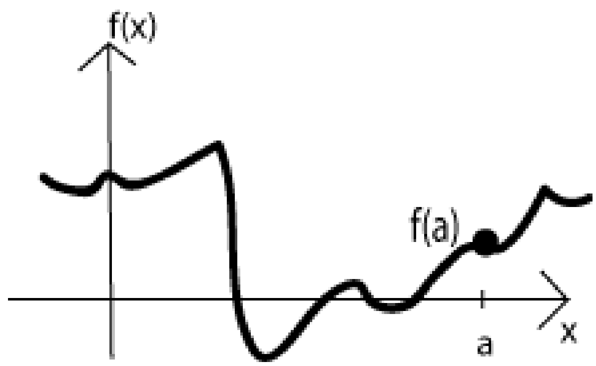


Figure 2.1

There are four ways to classify signals according to the values that the independent and dependent variables can take. Refer to the table below.

[1]This content is available online at <http://cnx.org/content/m11502/1.4/>.

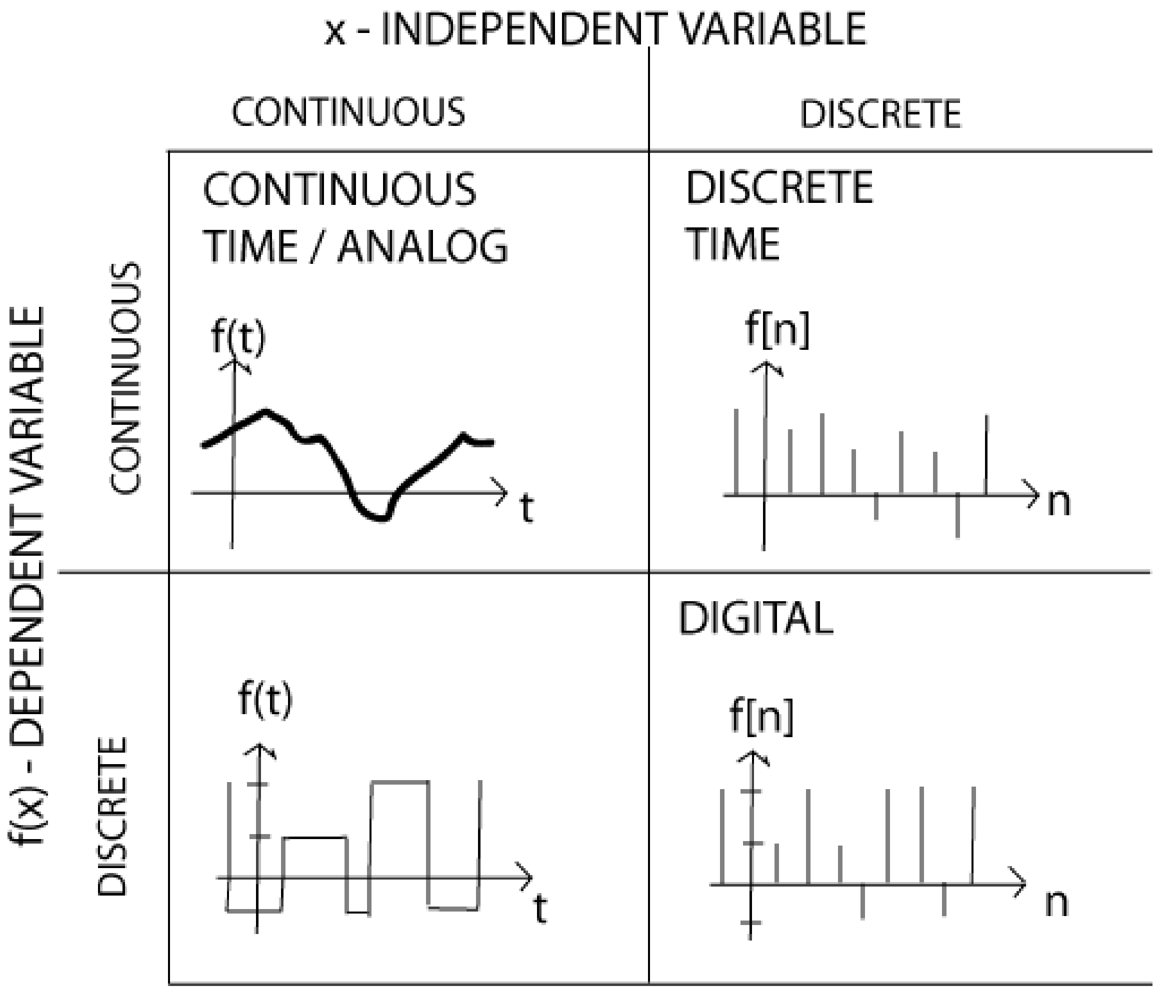


Figure 2.2

2.1.1.1 Quick Aside on Signal Notation

Continuous Time signals are represented as $f(t)$ where $t \in \mathbb{R}$. Discrete Time signals are represented as $f[n]$ where $n \in \mathbb{Z}$

2.1.2 The Four Fundamental Types of Signals[2]

2.1.2.1 Continuous-Time, Finite Length Signals

2.1.2.2 Continuous-Time, Infinite Length Signals

2.1.2.3 Discrete-Time, Finite Length Signals

2.1.2.4 Discrete-Time, Infinite Length Signals

2.1.3 Introduction to Sampling and Reconstruction[3]

Potential Existing modules: Sampling[4] Reconstruction[5]

2.2 Properties of Signals

2.2.1 Analog and Digital Signals[6]

2.2.2 Periodic Signals[7]

Recall that a periodic function is a function that repeats itself exactly after some given period, or cycle. We represent the definition of a **periodic function** mathematically as:

$$f(t) = f(t + mT) \forall m : (m \in \mathbb{Z}) \tag{2.1}$$

where $T > 0$ represents the **period**. Because of this, you may also see a signal referred to as a T-periodic signal. Any function that satisfies this equation is periodic.

We can think of periodic functions (with period T) two different ways:

#1) as functions on **all** of $\mathbb{R}$

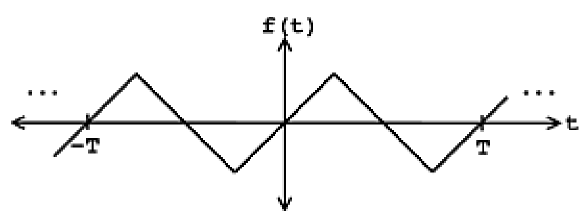


Figure 2.3: Function over all of $\mathbb{R}$ where $f(t_0) = f(t_0 + T)$

#2) or, we can cut out all of the redundancy, and think of them as functions on an interval $[0, T]$ (or, more generally, $[a, a + T]$). If we know the signal is T-periodic then all the information of the signal is captured by the above interval.

[2]This content is available online at <http://cnx.org/content/m11503/1.1/>.
[3]This content is available online at <http://cnx.org/content/m11530/1.1/>.
[4]"Sampling" <http://cnx.org/content/m10798/latest/>
[5]"Reconstruction" <http://cnx.org/content/m10788/latest/>
[6]This content is available online at <http://cnx.org/content/m11504/1.1/>.
[7]This content is available online at <http://cnx.org/content/m10744/2.7/>.

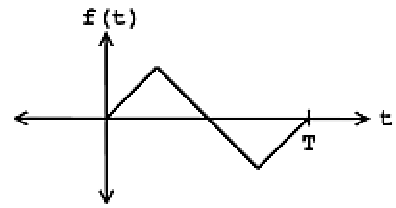


Figure 2.4: Remove the redundancy of the period function so that $f(t)$ is undefined outside $[0, T]$.

An **aperiodic** CT function $f(t)$ does not repeat for **any** $T \in \mathbb{R}$; *i.e.* there exists no T s.t. this equation (2.1) holds.

Question: DT definitions?

2.2.2.1 Continuous-Time

2.2.2.2 Discrete-Time

Note: Circular vs. Line

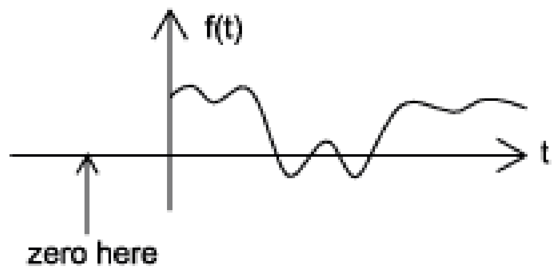
[Media Object][8]

2.2.3 Causal Signals[9]

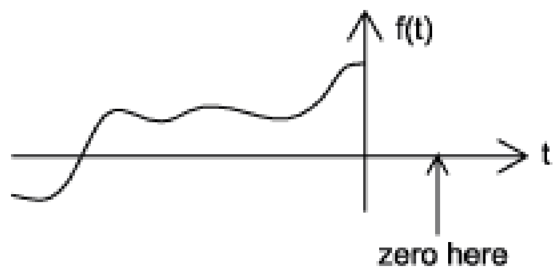
Causal signals are signals that are zero for all negative time, while **anitcausal** are signals that are zero for all positive time. **Noncausal** signals are signals that have nonzero values in both positive and negative time.

[8] This media object is a LabVIEW VI. Please view or download it at <PhaseShift.llb>

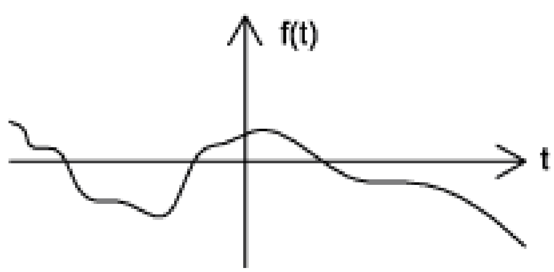
[9] This content is available online at <http://cnx.org/content/m11495/1.3/>.



(a)



(b)



(c)

Figure 2.5: (a) A causal signal (b) An anticausal signal (c) A noncausal signal

2.2.4 Real and Complex Signals[10]

A **real** signal $f(t)$ takes for each independent variable t a **real** value $f(t)$.

[10]This content is available online at <http://cnx.org/content/m11529/1.2/>.

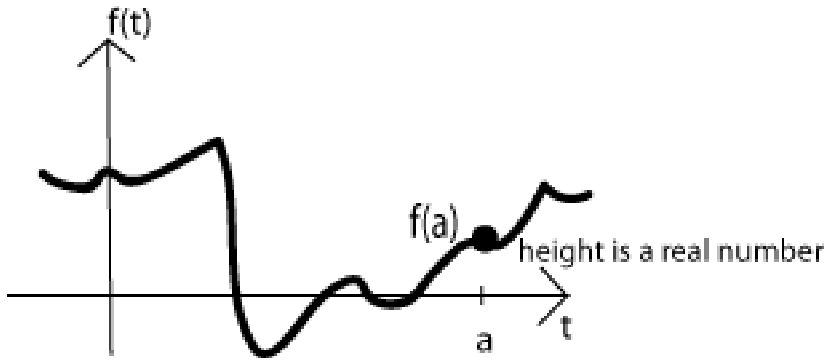


Figure 2.6

A **complex** signal $f(t)$ takes for each independent variable t a **complex** value $f(t)$.

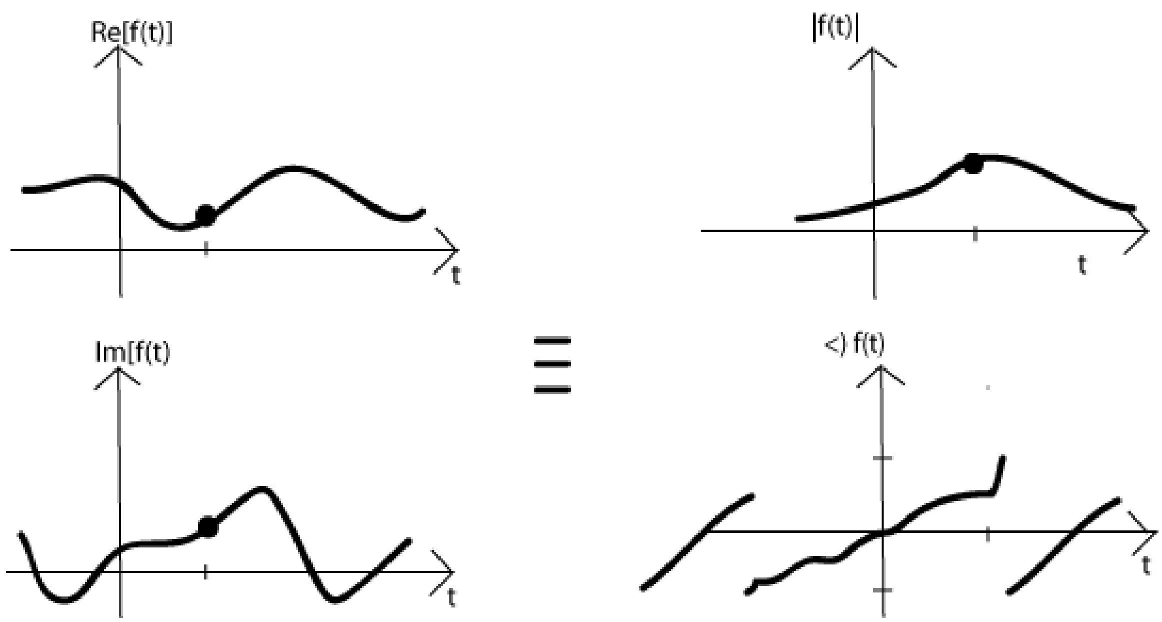


Figure 2.7

2.3 Important Signals

2.3.1 Delta Function - Heuristic Definition[11]

2.3.2 Delta Function as a Generalized Function[12]

Check out module m10170 - The Impulse Function

[11]This content is available online at <http://cnx.org/content/m11485/1.1/>.
[12]This content is available online at <http://cnx.org/content/m11484/1.3/>.

2.3.3 Sifting Property of the Delta Function[13]

2.3.4 Step Function in Continuous-Time[14]

2.3.5 Step Function in Discrete Time[15]

2.3.6 Sinusoids in Continuous Time[16]

2.3.6.1 Finite Length

2.3.6.2 Infinite Length

2.3.7 Sinusoids in Discrete Time[17]

2.3.7.1 Finite Length

2.3.7.2 Infinite Length

2.3.8 Sinc Function in Continuous Time[18]

2.3.9 Sinc Function in Discrete Time[19]

2.3.10 Complex Exponential in Continuous Time[20]

2.3.11 Complex Exponential in Discrete Time[21]

2.4 Size of a signal

2.4.1 Energy of a Signal[22]

2.4.1.1 Continuous Time Finite Length"

2.4.1.2 Continuous Time Infinite Length

2.4.1.3 Discrete Time Finite Length"

2.4.1.4 Discrete Time Infinite Length

2.4.2 Norm of a Signal[23]

2.4.2.1 Continuous Time Finite Length"

2.4.2.2 Continuous Time Infinite Length

2.4.2.3 Discrete Time Finite Length"

2.4.2.4 Discrete Time Infinite Length

2.4.3 Power of a signal[24]

2.4.3.1 Continuous Time"

2.4.3.2 Discrete Time"

[13] This content is available online at <http://cnx.org/content/m11508/1.1/>.
[14] This content is available online at <http://cnx.org/content/m11486/1.2/>.
[15] This content is available online at <http://cnx.org/content/m11505/1.1/>.
[16] This content is available online at <http://cnx.org/content/m11488/1.3/>.
[17] This content is available online at <http://cnx.org/content/m11489/1.3/>.
[18] This content is available online at <http://cnx.org/content/m11506/1.1/>.
[19] This content is available online at <http://cnx.org/content/m11507/1.1/>.
[20] This content is available online at <http://cnx.org/content/m11490/1.1/>.
[21] This content is available online at <http://cnx.org/content/m11491/1.1/>.
[22] This content is available online at <http://cnx.org/content/m11509/1.1/>.
[23] This content is available online at <http://cnx.org/content/m11409/1.4/>.
[24] This content is available online at <http://cnx.org/content/m11510/1.2/>.

Chapter 3

Operators

3.1 Linearity

3.1.1 Scaling and Superposition[1]

3.1.1.1 Continuous Time

Add VI

3.1.1.2 Discrete Time

Add VI

3.1.2 Linear Operators[2]

3.1.2.1 Continuous Time

add VI

3.1.2.2 Discrete Time

add VI

3.1.3 Characterization of Linear Operators[3]

3.1.3.1 Continuous Time

link to 41

3.1.3.2 Discrete Time

Link to 41

[1]This content is available online at <http://cnx.org/content/m11512/1.1/>.
[2]This content is available online at <http://cnx.org/content/m11513/1.1/>.
[3]This content is available online at <http://cnx.org/content/m11514/1.1/>.

3.1.4 Linearity/Nonlinearity Examples[4]

3.1.4.1 Continuous Time

3.1.4.2 Discrete Time

3.2 Time Invariance

3.2.1 Time Invariant Operators[5]

3.2.2 Time Invariant/Time Variant Examples[6]

3.3 LTI Operators[7]

3.4 Characterizationi of LTI Operators[8]

3.4.1 Continuous Time

Characterization of Linear Operators (Section 3.1.3) Time Invariant Operators (Section 3.2.1)

3.4.2 Discrete Time

add VI

3.5 Discrete Time System Analysis

3.5.1 Discrete Time Signals are Vectors[9]

3.5.1.1 Finite Length Signals

The Four Fundamental types of Signals (Section 2.1.2)

3.5.1.2 Infinite Length Signals

The Four Fundamental types of Signals (Section 2.1.2)

Time Variant/Time Invariant Examples (Section 3.2.2) Step Function in Discrete Time (Section 2.3.5) Sinc Function in Discrete Time (Section 2.3.9) Sinusoids in Discrete Time (Section 2.1.2) The Complex Exponential in Discrete Time (Section 2.1.2)

3.5.2 Discrete Time Linear Systems are Matrices[10]

3.5.2.1 Finite Length

Characterization of Linear Operators (Section 3.1.3)

3.5.2.2 Infinite Length

Characterization of Linear Operators (Section 3.1.3)

[4]This content is available online at <http://cnx.org/content/m11515/1.2/>.
[5]This content is available online at <http://cnx.org/content/m11516/1.1/>.
[6]This content is available online at <http://cnx.org/content/m11517/1.1/>.
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[10]This content is available online at <http://cnx.org/content/m11522/1.1/>.

3.5.3 Discrete Time LTI systems as Matrices[11]

3.5.3.1 Finite Length

circulant Matrices

3.5.3.2 Infinite Length

Toeplitz Matrices

3.5.4 Impulse Response of a Linear Discrete Time System[12]

3.5.4.1 Finite Length

3.5.4.2 Infinite Length

Sifting Property of Delta Function (Section 2.3.3)

3.5.5 Impulse Response of an LTI Discrete Time System[13]

3.5.5.1 Finite Length

3.5.5.2 Infinite Length

3.5.6 Convolution

3.5.6.1 Discrete-Time Convolution[14]

3.5.6.1.1 Overview

Convolution is a concept that extends to all systems that are both **linear and time-invariant (Section 1.3)** (**LTI**). The idea of **discrete-time convolution** is exactly the same as that of continuous-time convolution[15]. For this reason, it may be useful to look at both versions to help your understanding of this extremely important concept. Recall that convolution is a very powerful tool in determining a system's output from knowledge of an arbitrary input and the system's impulse response. It will also be helpful to see convolution graphically with your own eyes and to play around with it some, so experiment with the applets[16] available on the internet. These resources will offer different approaches to this crucial concept.

3.5.6.1.2 Convolution Sum

As mentioned above, the convolution sum provides a concise, mathematical way to express the output of an LTI system based on an arbitrary discrete-time input signal and the system's response. The **convolution sum** is expressed as

$$y[n] = \sum_{k=-\infty}^{\infty} (x[k]\, h[n-k]) \tag{3.1}$$

As with continuous-time, convolution is represented by the symbol *, and can be written as

$$y[n] = x[n] * h[n] \tag{3.2}$$

[11]This content is available online at <http://cnx.org/content/m11523/1.1/>.
[12]This content is available online at <http://cnx.org/content/m11524/1.1/>.
[13]This content is available online at <http://cnx.org/content/m11525/1.1/>.
[14]This content is available online at <http://cnx.org/content/m10087/2.19/>.
[15]"Continuous-Time Convolution" <http://cnx.org/content/m10085/latest/>
[16]http://www.jhu.edu/~signals

By making a simple change of variables into the convolution sum, $k = n - k$, we can easily show that convolution is **commutative**:

$$x[n] * h[n] = h[n] * x[n] \quad (3.3)$$

For more information on the characteristics of convolution, read about the Properties of Convolution[17].

3.5.6.1.3 Derivation

We know that any discrete-time signal can be represented by a summation of scaled and shifted discrete-time impulses. Since we are assuming the system to be linear and time-invariant, it would seem to reason that an input signal comprised of the sum of scaled and shifted impulses would give rise to an output comprised of a sum of scaled and shifted impulse responses. This is exactly what occurs in **convolution**. Below we present a more rigorous and mathematical look at the derivation:

Letting $\mathcal{H}$ be a DT LTI system, we start with the following equation and work our way down the convolution sum!

$$\begin{aligned} y[n] &= \mathcal{H}[x[n]] \\ &= \mathcal{H}\left[\sum_{k=-\infty}^{\infty} (x[k]\delta[n-k])\right] \\ &= \sum_{k=-\infty}^{\infty} (\mathcal{H}[x[k]\delta[n-k]]) \\ &= \sum_{k=-\infty}^{\infty} (x[k]\mathcal{H}[\delta[n-k]]) \\ &= \sum_{k=-\infty}^{\infty} (x[k]h[n-k]) \end{aligned} \quad (3.4)$$

Let us take a quick look at the steps taken in the above derivation. After our initial equation, we using the DT sifting property[18] to rewrite the function, $x[n]$, as a sum of the function times the unit impulse. Next, we can move around the $\mathcal{H}$ operator and the summation because $\mathcal{H}[\cdot]$ is a linear, DT system. Because of this linearity and the fact that $x[k]$ is a constant, we can pull the previous mentioned constant out and simply multiply it by $\mathcal{H}[\cdot]$. Finally, we use the fact that $\mathcal{H}[\cdot]$ is time invariant in order to reach our final state - the convolution sum!

A quick graphical example may help in demonstrating why convolution works.

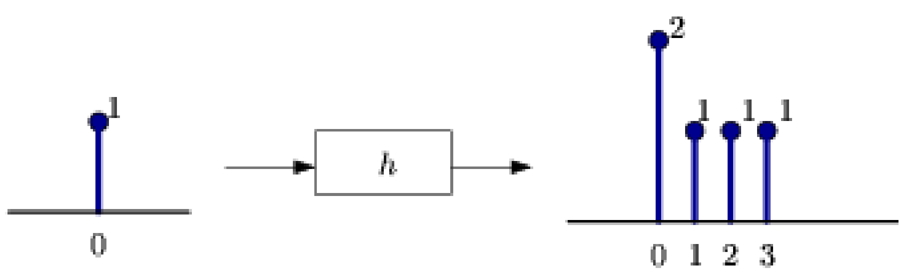

Figure 3.1: A single impulse input yields the system's impulse response.

[17]"Properties of Convolution" <http://cnx.org/content/m10088/latest/>
[18]"The Impulse Function": Section The Sifting Property of the Impulse <http://cnx.org/content/m10059/latest/#sifting>

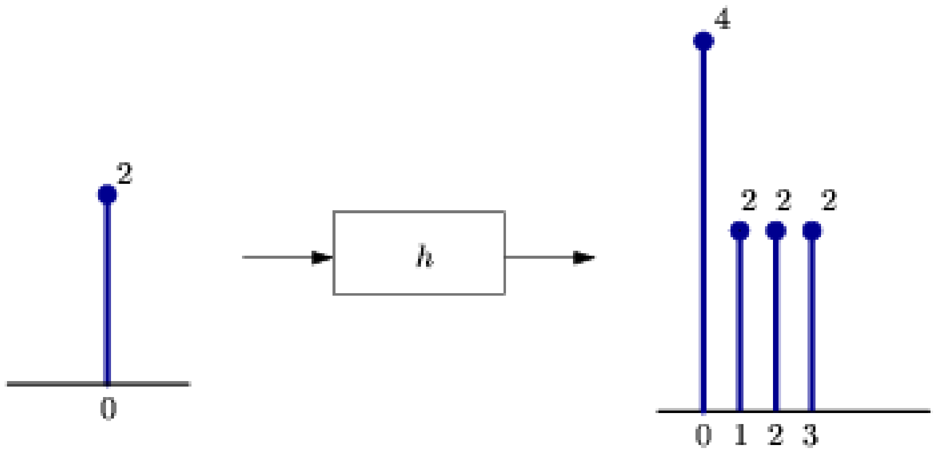

Figure 3.2: A scaled impulse input yields a scaled response, due to the scaling property of the system's linearity.

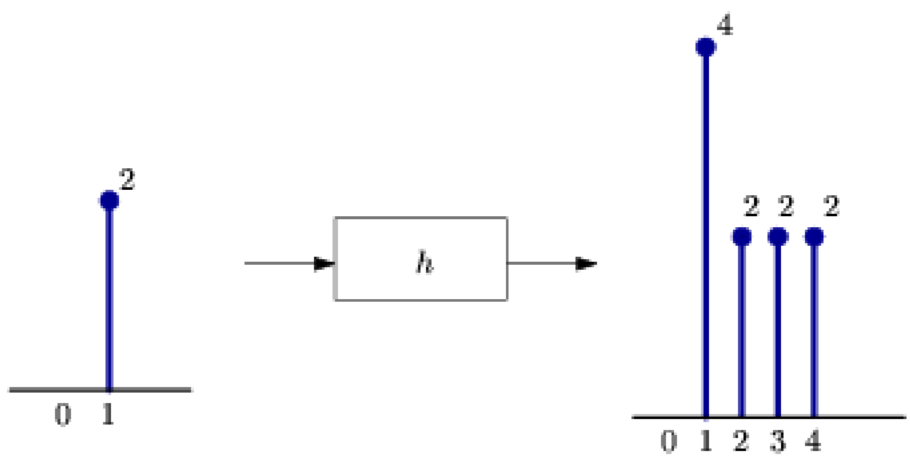

Figure 3.3: We now use the time-invariance property of the system to show that a delayed input results in an output of the same shape, only delayed by the same amount as the input.

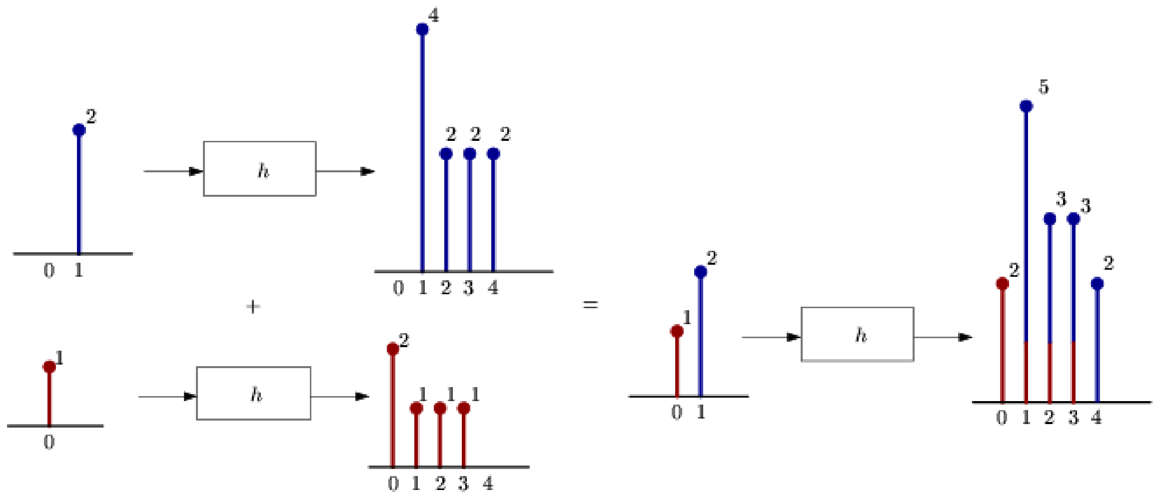

Figure 3.4: We now use the additivity portion of the linearity property of the system to complete the picture. Since any discrete-time signal is just a sum of scaled and shifted discrete-time impulses, we can find the output from knowing the input and the impulse response.

3.5.6.1.4 Convolution Through Time (A Graphical Approach)

In this section we will develop a second graphical interpretation of discrete-time convolution. We will begin this by writing the convolution sum allowing x to be a causal, length-m signal and h to be a causal, length-k, LTI system. This gives us the finite summation,

$$y[n] = \sum_{l=0}^{m-1} (x[l]\, h[n-l]) \qquad (3.5)$$

Notice that for any given n we have a sum of the products of x_l and a time-delayed h_{-l}. This is to say that we multiply the terms of x by the terms of a time-reversed h and add them up.

Going back to the previous example:

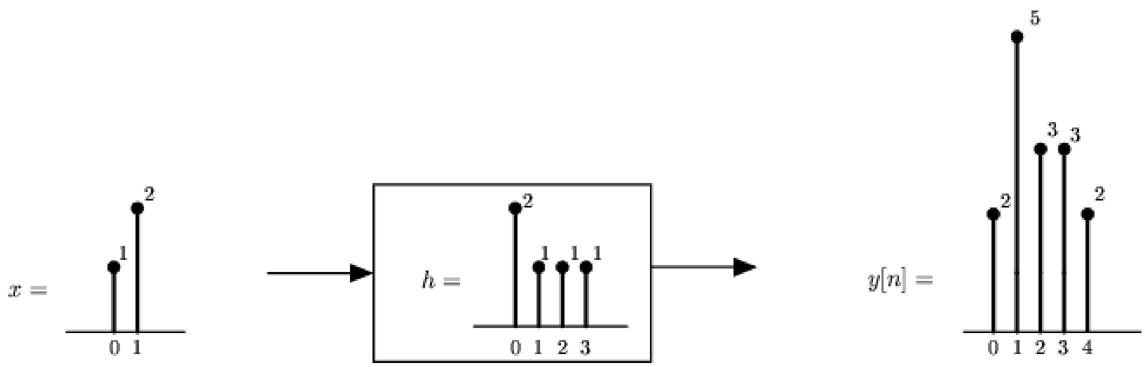

Figure 3.5: This is the end result that we are looking to find.

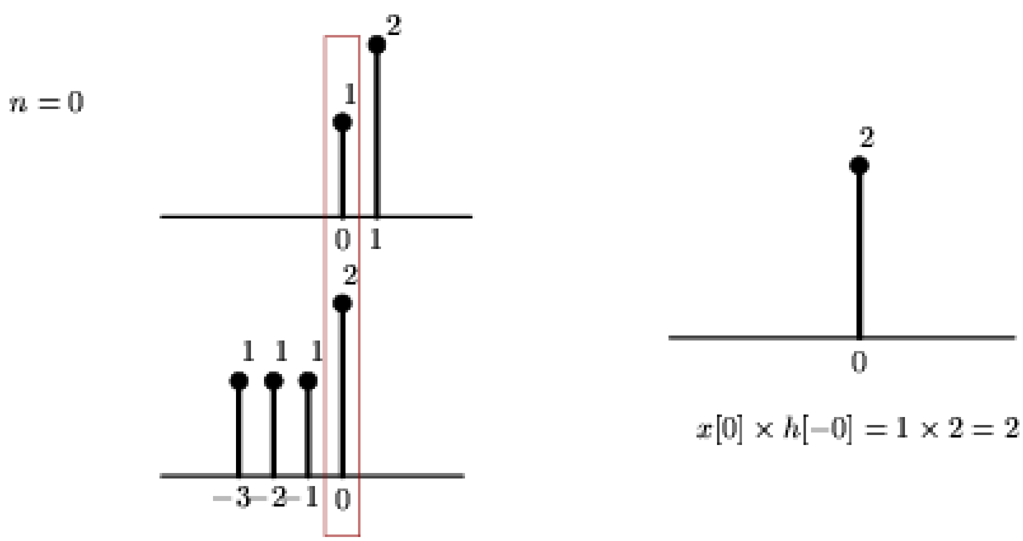

Figure 3.6: Here we reverse the impulse response, h , and begin its traverse at time 0.

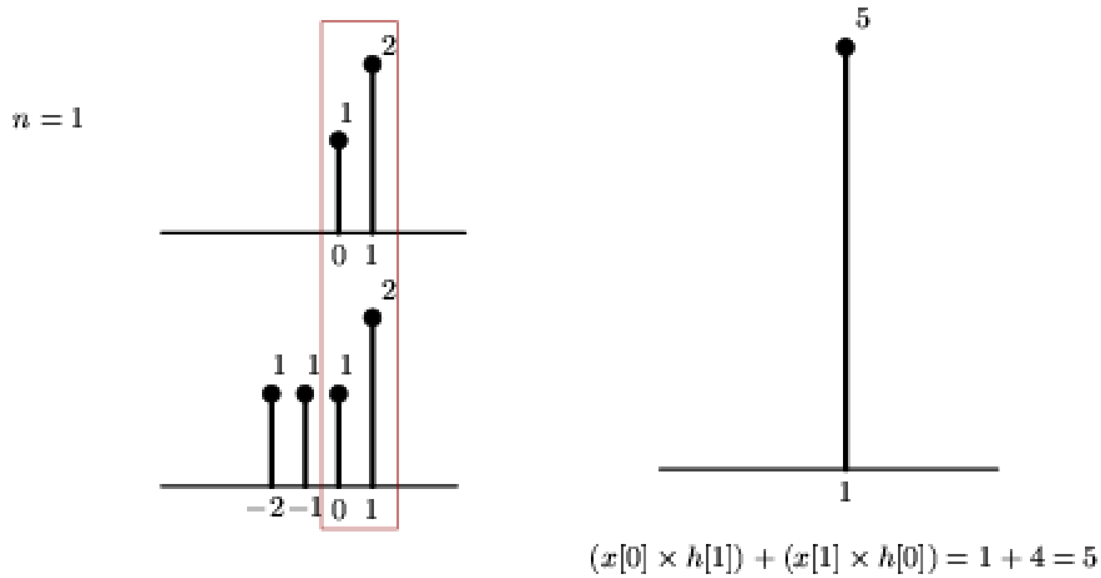

Figure 3.7: We continue the traverse. See that at time 1 , we are multiplying two elements of the input signal by two elements of the impulse response.

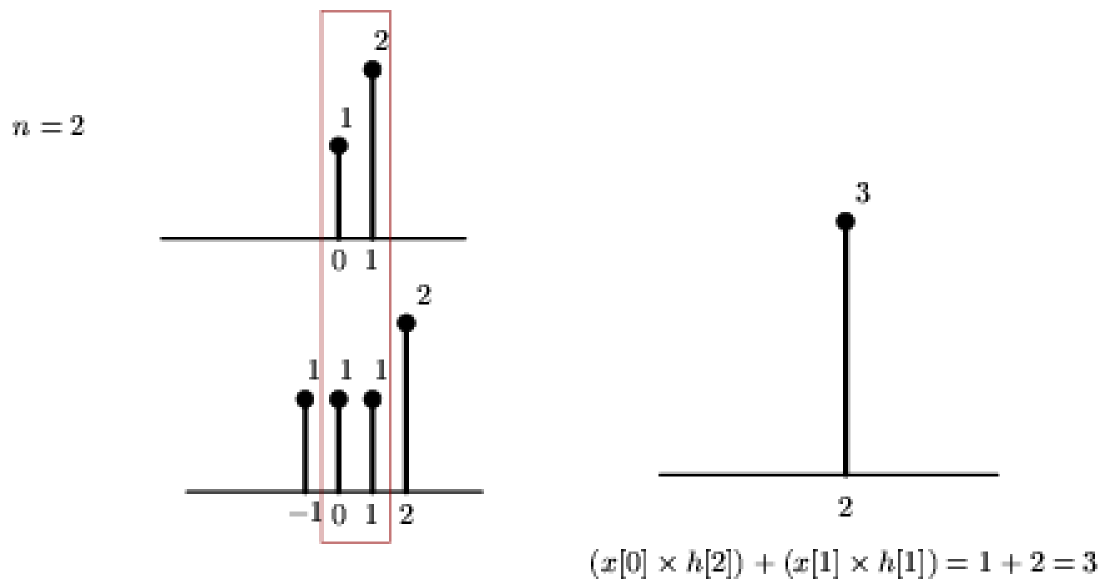

Figure 3.8

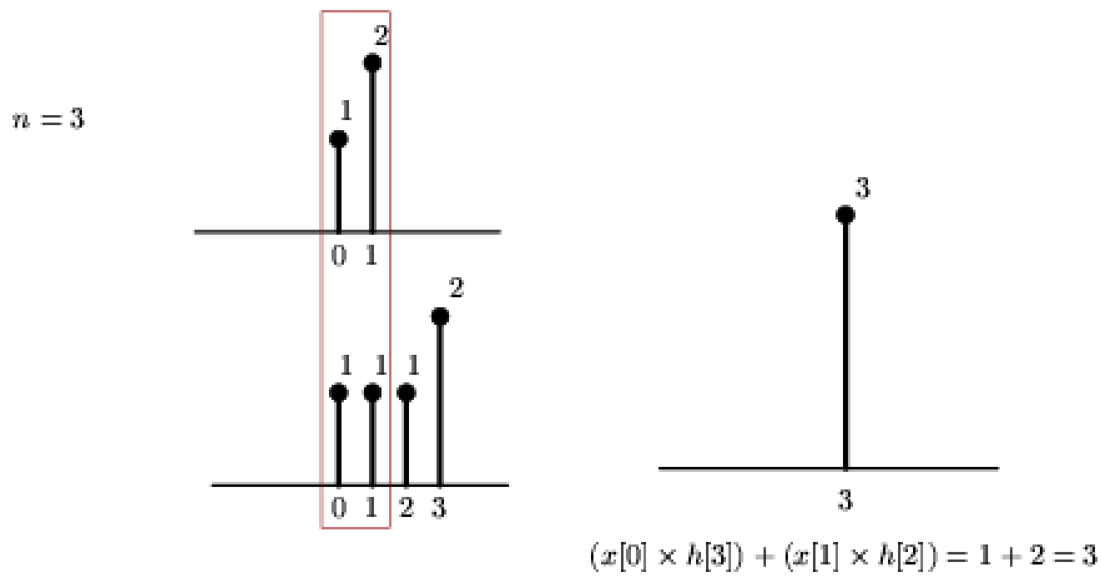

Figure 3.9: If we follow this through to one more step, $n = 4$, then we can see that we produce the same output as we saw in the initial example.

What we are doing in the above demonstration is reversing the impulse response in time and "walking it across" the input signal. Clearly, this yields the same result as scaling, shifting and summing impulse responses.

This approach of time-reversing, and sliding across is a common approach to presenting convolution, since it demonstrates how convolution builds up an output through time.

Chapter 4

Appendix

4.1 Complex numbers and Arithmetic[1]

4.2 Riemann Integration[2]

[1]This content is available online at <http://cnx.org/content/m11497/1.1/>.
[2]This content is available online at <http://cnx.org/content/m11511/1.2/>.

Index of Keywords and Terms

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