Notes on Basic Mechanics for Rice ELEC 201

By:

Jim Young

Notes on Basic Mechanics for Rice ELEC 201

By:

Jim Young

Online:
< http://cnx.org/content/col10357/1.1/ >

CONNEXIONS

Rice University, Houston, Texas

Collection structure revised: June 12, 2006
PDF generated: October 30, 2009
For copyright and attribution information for the modules contained in this collection, see p. 27.

Table of Contents

Introduction to Basic Mechanics 1
Forces 3
Torque 9
Simple Machine Elements 11
Springs 19
Counterweights 23

1 Source Information

1.1 Rice ELEC 201 Course Notes Project 25
Solutions .... **??**
Index 26
Attributions 27

Introduction to Basic Mechanics[1]

The branch of physics that deals with the action of forces on matter is referred to as **mechanics**. All considerations of motion are addressed by mechanics, as well as the transmission of forces through the use of simple machines. When building a small robot, the primary goal is mechanical, e.g., placing blocks into a bin, and electronics are used to control the mechanics.

While it is not necessary to sit down and draw free body diagrams or figure out the static coefficient of friction between the LEGO tires and the game board, it is helpful to keep certain mechanical concepts in mind when constructing a robot. If a robot's tires are spinning because they do not grip the floor, then something must be done to increase the friction between the tires and the floor. One solution is to glue a rubber band around the circumference of the tire. That problem/solution did not require an in-depth study of physics. Simply considering the different possibilities can lead to more mechanically creative robots.

Describing motion involves more than just saying that an object moved three feet to the right. The magnitude and direction of the displacement are important, but so are the characteristics of the object's velocity and acceleration. To understand these concepts, we must examine the nature of force. Changes in the motion of an object are created by forces.

---

[1] This content is available online at <http://cnx.org/content/m13534/1.1/>.

Forces[2]

An object in motion, or at rest, will not change its state of motion unless a force is applied. This resistance to changes in motion is called **inertia**. To be clear, a change in motion is not just beginning to move from a stop. Slowing down, speeding up, and changing direction are all changes in motion. The only way to change a object's motion is to apply a force to that object. A book slid across a table only comes to a stop because of the frictional forces acting on it. Inertia is proportional to mass, so a more massive object is more difficult to move or stop than a lighter one, even on a frictionless surface. This module will consider forces and friction, which both act on an object's inertia.

Just as a book slides until a force opposes its motion, a disc will spin until its rotation is opposed by some force. This property is aptly named **rotational inertia**. One of the most common applications of rotational inertia is shown in . Many children's toys use rotational inertia. In friction-drive cars, the child pushes the car forward several times to set an internal flywheel in motion. When the car is put down, the flywheel is still spinning and the car moves. This is an interesting way to store energy – in kinetic, rather than potential form. A flywheel could conceivably be used to store energy to keep small robot operating after its motors were required to be shut off. Rotational inertia is also used to avoid changes in motion for such objects as record players, where it is important to maintain rotation at a constant speed.

---

[2] This content is available online at <http://cnx.org/content/m13535/1.2/>.

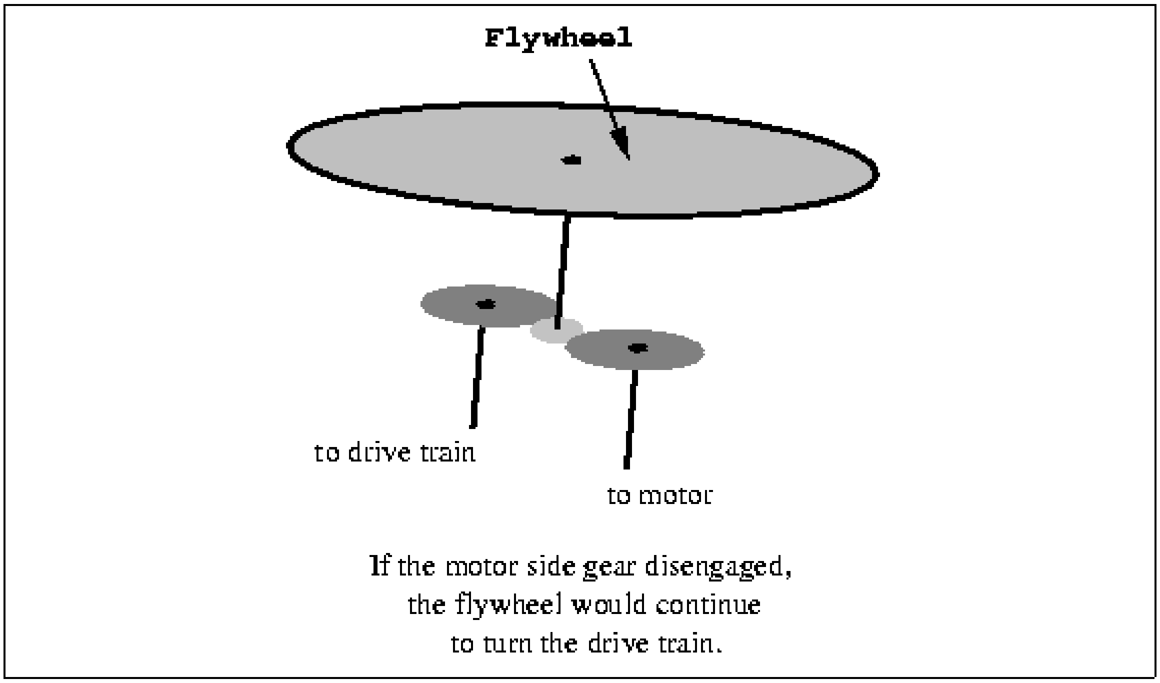

Figure 1: Flywheel

Force

Whether a force is the push of a motor or the pull of gravity, the important characteristics are the magnitude and direction of the force, and the mass and previous state of motion of the object being affected. By pushing on a moving car, one can either cause it to gain speed or come to a stop, depending on which direction the force is applied, and that same force applied to a feather would be expected to more drastically affect the motion of the feather.

It is common practice to determine the expected changes in motion that an object will experience due to a particular force with the aid of a **free body diagram**. A diagram can tell us at a glance in which direction we would expect an object to accelerate or decelerate. A free body diagram shows all of the forces acting on an object, even if their effects are balanced out by another force. We will use free body diagrams to consider different situations involving the lamp that you find at your lab station (Figure 2).

One force that always acts on the lamp is gravity. This familiar force would accelerate the lamp downward toward the center of the earth **if** left unchallenged. However, when the lamp is placed on a table it does not move downward because the table holds it up. The lamp is pushing down on the table and the table is pushing up on the lamp. This pair of forces is an action-reaction pair: equal and opposite forces acting on two different objects in contact. The reaction force from the table is called the **normal force** because this force is oriented normal (perpendicular) to the surface of the table. The arrows representing the forces are labeled. The symbols over the labels remind us that the forces are vector quantities and that the direction in which the force is applied is important. The length of the force vector should be proportional to their magnitudes.

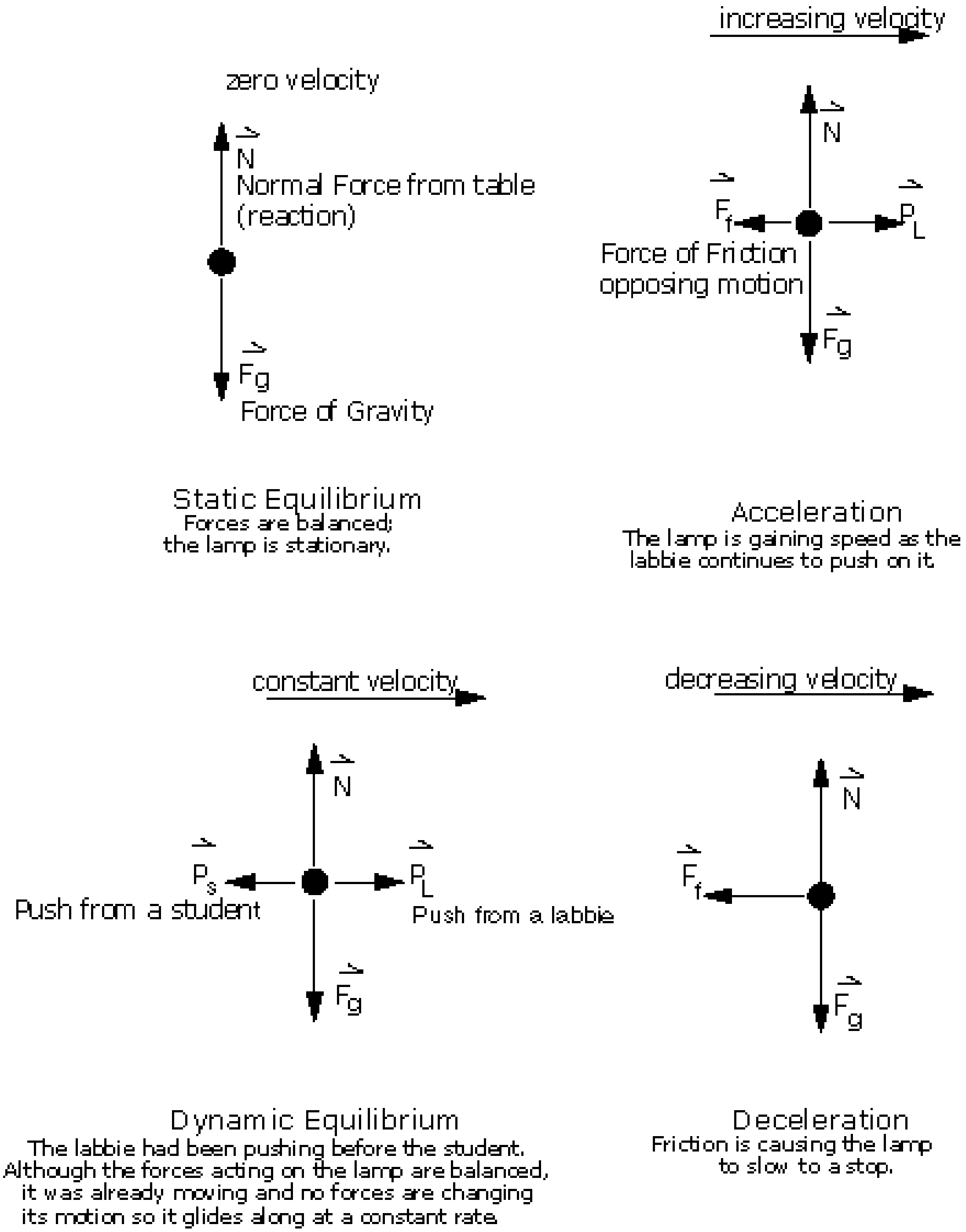

Figure 2: Free Body Diagrams

In Figure 2 the lamp was represented by a simple dot. We assumed that the lamp was rigid and that a downward force applied at one particular spot on the lamp would yield the same result as a similar downward force applied at a different place on the lamp. Actually, in order for a force of equal magnitude and direction to affect an object's motion in the same manner it must be applied along the same line of action as the original force (see Figure 3). If the original force had been a tug on a string tied to the lamp, then it makes sense that grabbing the string at a different distance away from the lamp to tug should not make a difference provided that the direction and magnitude do not change.

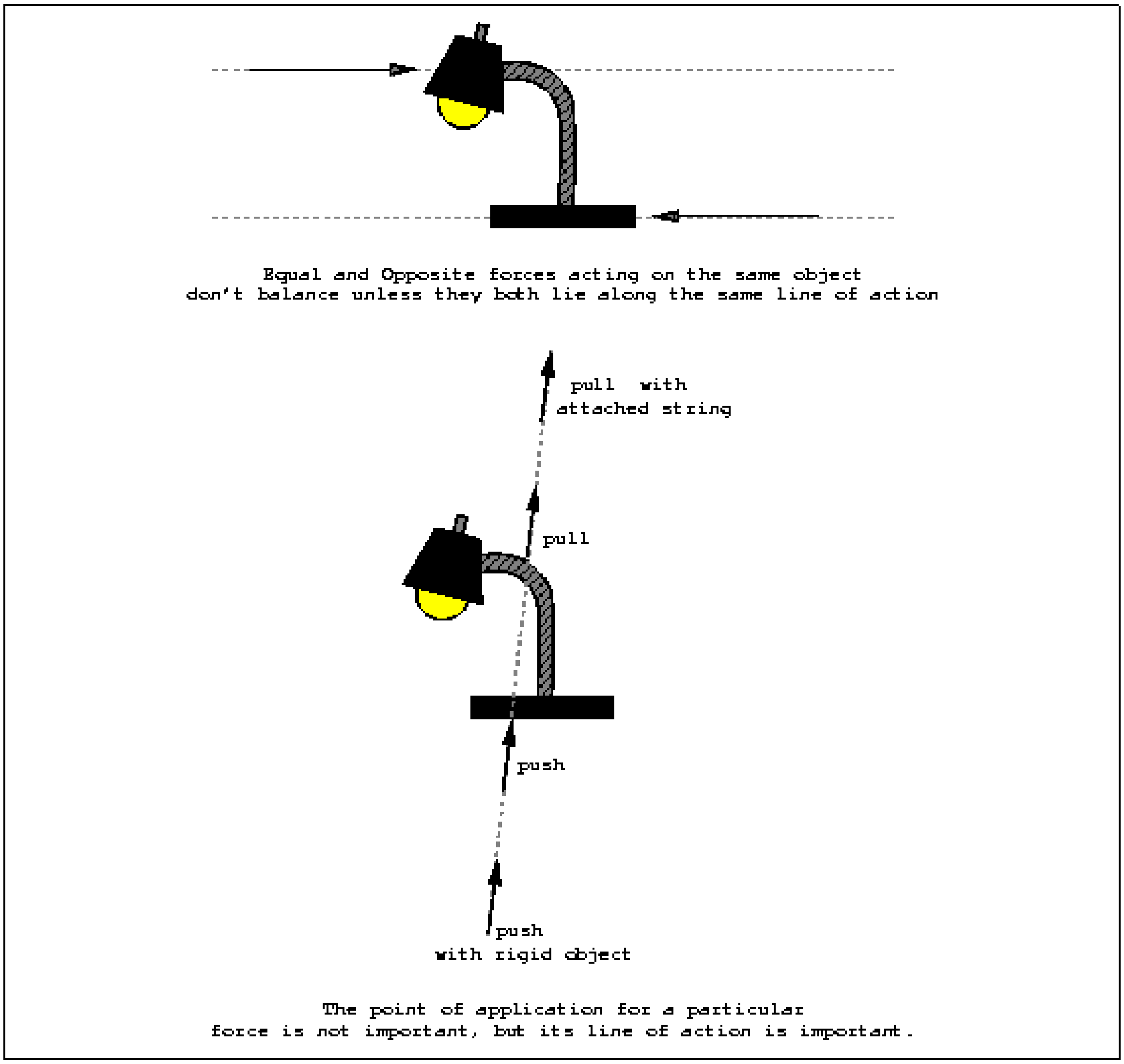

Figure 3: Line of Action

Friction

The normal force from the table's surface is a reaction force **only**. Without the downward force on the table from the object resting its weight on the surface, the normal force does not exist. This type of behavior is also descriptive of frictional forces.

Friction is opposition to motion, so if nothing is trying to move there will be no friction. However, friction will be present when motion is attempted, even if the object is not yet moving. There are two different types of friction: static, which acts before the object begins to move, and dynamic, which acts after the object begins moving. **Static friction** is usually stronger than **dynamic friction**.

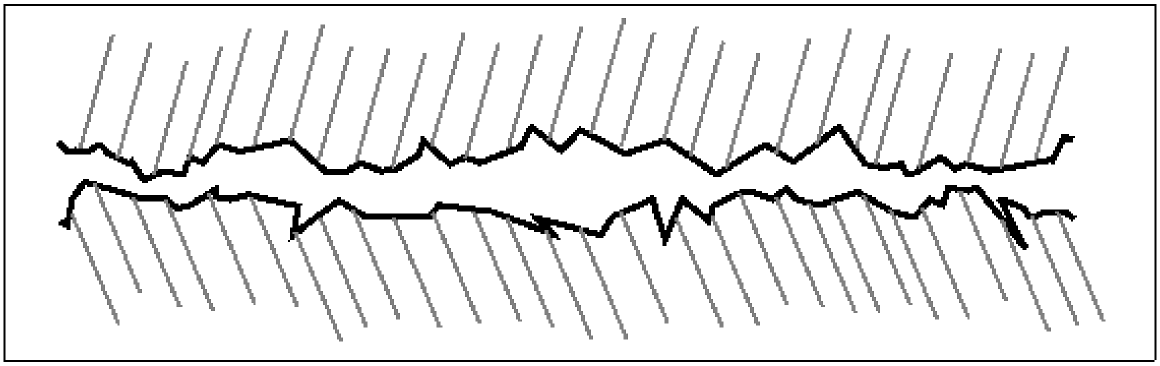

Figure 4: "Close Up" of surfaces in contact

Friction occurs because the surfaces in contact are **not** smooth. The small ridges on the different surfaces catch, and in order for the objects to move, these ridges must be broken off or the object must ramp up and over the obstructions. By adding a lubricant between the two layers, it is possible to "float" one layer high enough to miss some of the obstructions to motion. At an atomic level, cold joints may form where the atoms from one object's surface may form weak bonds with the atoms on the surface of the other object. These bonds must also be broken in order for the object to move. All of this resistance to motion is called **friction**. Friction is very important because it not only inhibits motion, friction also makes motion possible.

Most, but not all, small robots (such as those built in the Rice University course ELEC 201, Introduction to Engineering Design) will probably be wheeled vehicles, and without friction those wheels would just spin in place without moving the robot anywhere. In order to increase the friction between the wheels and the game board one might use wheels made of a different material or add a rubber band around the wheel's circumference. Friction is not desirable in all cases. When it comes to axles spinning inside of holes in beams or gears rubbing up against beams or even gears pushing against each other, friction can cause two identically constructed gear trains to behave differently. Friction can even render the whole assembly ineffective. For example, in one design, a worm gear in a drive train created so much friction that more of the drive motor's effort went towards overcoming friction than actually driving the robot.

Torque[3]

A **torque** is a force applied at a distance from a pivot point, such as an axle or shaft. When describing torques, one must include the magnitude and direction of the force, plus the perpendicular distance from the pivot where the force is applied. The magnitude of the torque is the **product** of the force and the distance from the pivot. Since torque is a product of force and distance, one may be "traded" for the other. The same torque can be created by applying a different force at a different distance from the pivot.

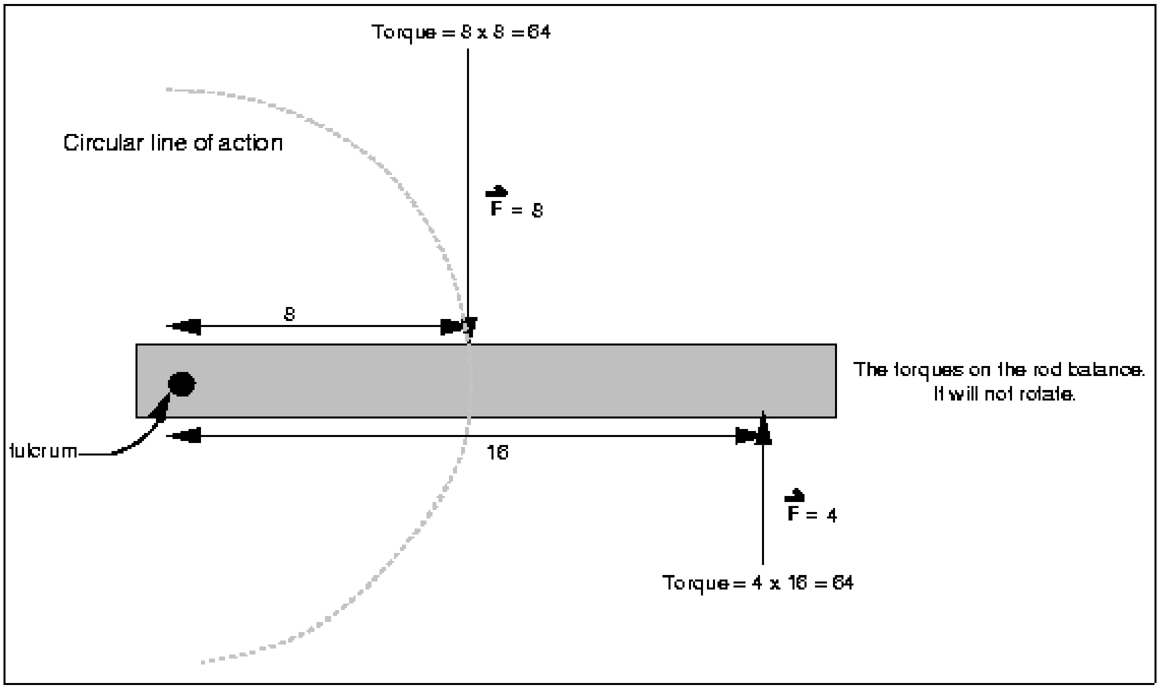

Figure 1: Illustrating Torque

Figure 1 shows a beam, such as a LEGO beam, with an axle pivot at one end, the fulcrum. The beam is free to rotate about the fulcrum, so a downward force will cause the beam to rotate downward, or clockwise, in Figure 1. The force of 8 units shown applied at a distance of 8 units from the fulcurm produces a downward, or clockwise, torque of 64 units.

---

[3] This content is available online at <http://cnx.org/content/m13544/1.1/>.

If we want the beam to remain stationary, we must apply an equal torque in the upward direction. We can apply force at many points along the beam, but the force needed depends on where we apply it because of the force-distance trade-off of torque. We could apply an upward force at the same place as the downward force, having the same magnitude. Or, as shown, we could apply a smaller force, 4 units, at a larger distance, 16 units. The smaller force at a larger distance produces the same torque, 64 units. By the same reasoning, we would have to apply a larger force closer to the pivot to produce the same upward torque.

Figure 1 illustrates how a simple lever works and why it is useful. Applying a small force at a large distance from the pivot can produce or oppose a very large force close to the pivot. "Give me a long enough lever, and I can move the world."

In summary, **torque** is a force applied at a distance from a pivot. When describing torques, one must include magnitude, direction, and perpendicular distance from the pivot. Since torque is a **product** of force and distance, one may be "traded" for the other. The concept of trading distance traveled/applied for force experienced/applied is key to many simple machines.

Simple Machine Elements[4]

Complex machines are made up of moving parts such as levers, gears, cams, cranks, springs, belts, and wheels. Machines deliver a certain type of movement to a desired location from an input force applied somewhere else. Some machines simply convert one type of motion to another type, such as rotary to linear. While there is a seemingly endless variety of machines, they are all based upon simple machine elements. The elements discussed here include inclined planes, levers, wheels and axles, pulleys, and screws.

It is important to remember that all machines are limited in their efficiency by friction. No machine is 100 percent efficient in its efforts, so the mechanical advantage gained will require additional energy to accomplish the task. For more information on friction, see this module[5]

The Inclined Plane

An **inclined plane** decreases the force required to raise an object a given height by increasing the distance over which that force must be applied, see Figure 1. Imagine lifting something twice your weight to a 4 foot high shelf. Now imagine rolling the same mass up a gently sloping surface. The latter would be much easier. Inclined planes are commonly put to use in cutting devices and often two inclined planes are put back to back to form a wedge. In a wedge, forward movement is converted into a parting movement acting perpendicular to the face of the blade. A zipper is simply a combination of two lower wedges for closing and an upper wedge for opening, as shown in Figure 2.

---

[4] This content is available online at <http://cnx.org/content/m13594/1.1/>.

[5] "Introduction" <http://cnx.org/content/m11106/latest/>

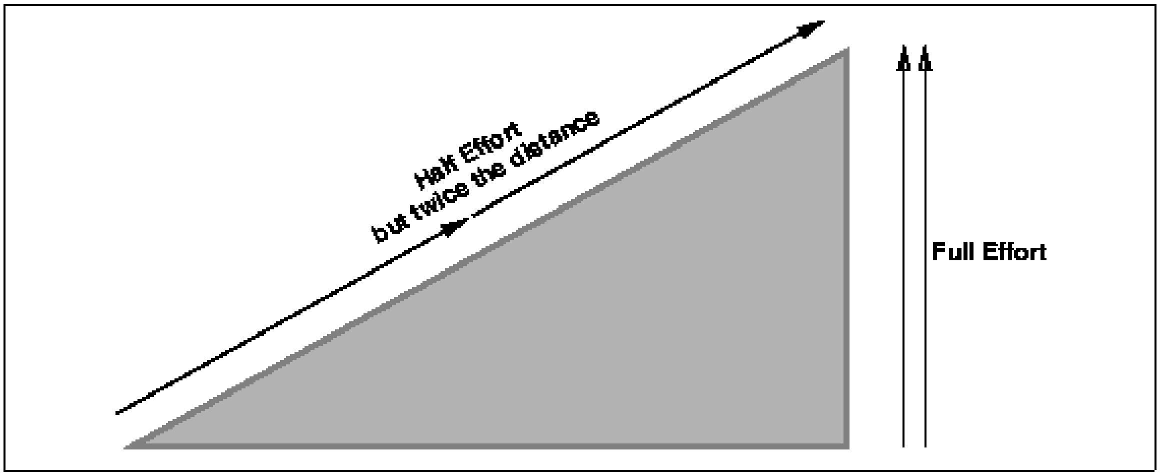

Figure 1: Inclined Plane

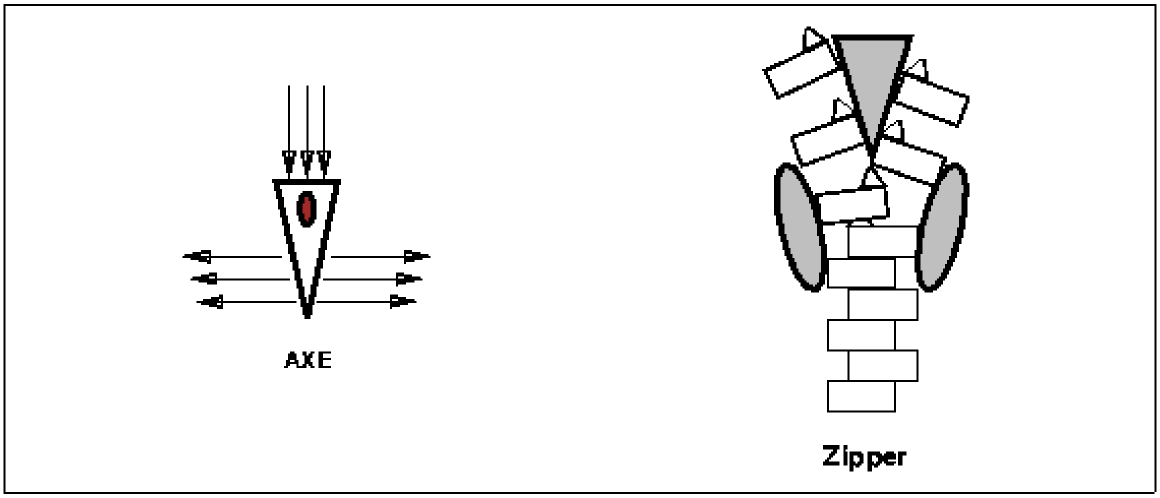

Figure 2: The Inclined Plane at Work

The Screw

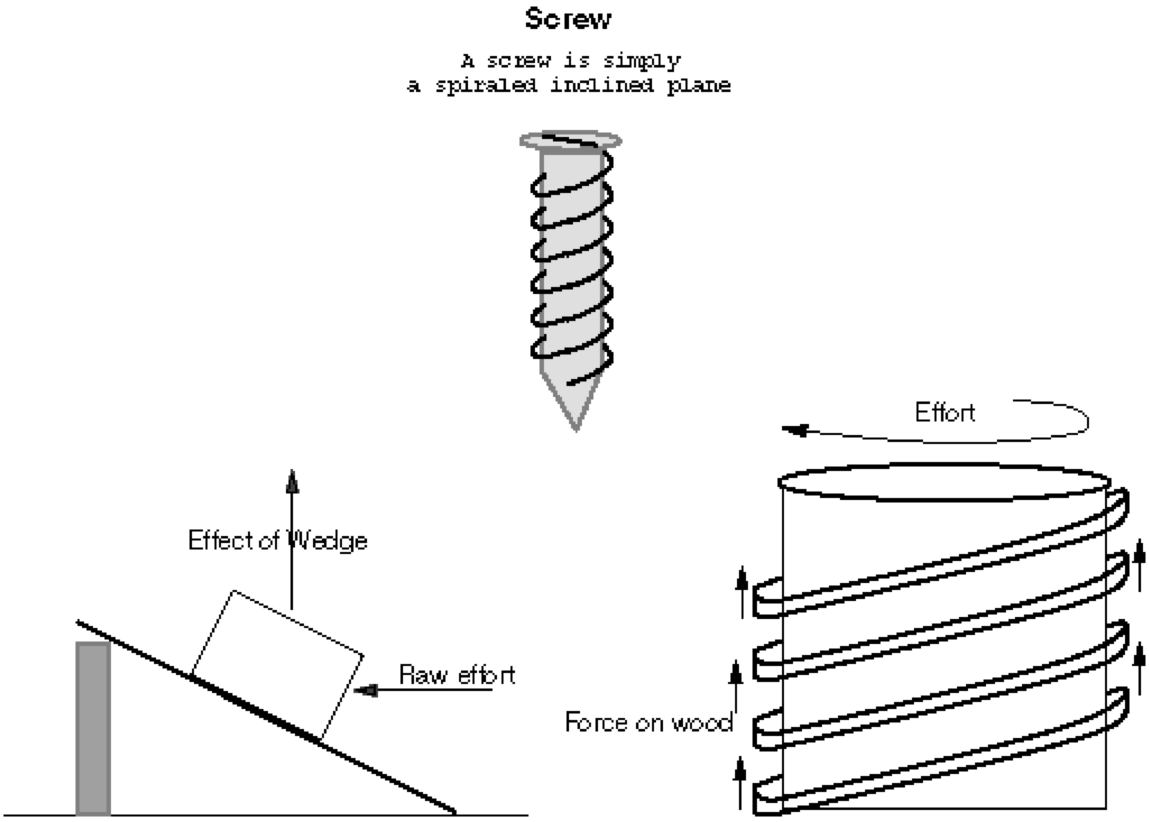

Figure 3: The Screw

The **screw** is basically an inclined plane (see Figure 3) wrapped around a cylinder. In an inclined plane, a linear force in the horizontal plane is converted to a vertical "lifting" force. With a screw, a rotary force in the horizontal plane is converted to a vertical "lifting" force.

When a wood screw is turned, the threads of the screw push up on the wood. A reaction force from the wood pushes back down on the screw threads and in this way the screw moves down even though the force of turning the screw is in the horizontal plane. Screws are known for high friction, which is why they are used to hold things together. A worm gear is sometimes used in machines, but they also have high friction that can waste considerable power.

Levers

A **lever** has three points of interest: the fulcrum, the load, and the effort applied to the lever. The **fulcrum** is the point around which the lever pivots rotationally. The **load** is what we wish to manipulate with the lever, and the load is described by its position relative to the fulcrum, and the force (magnitude and direction) it exerts at that point. The **effort** is also a force that has a magnitude and a direction, and a position with respect to the fulcrum. A lever is used to change the direction of movement, and to trade the magnitude of the effort for the distance over which the effort is applied.

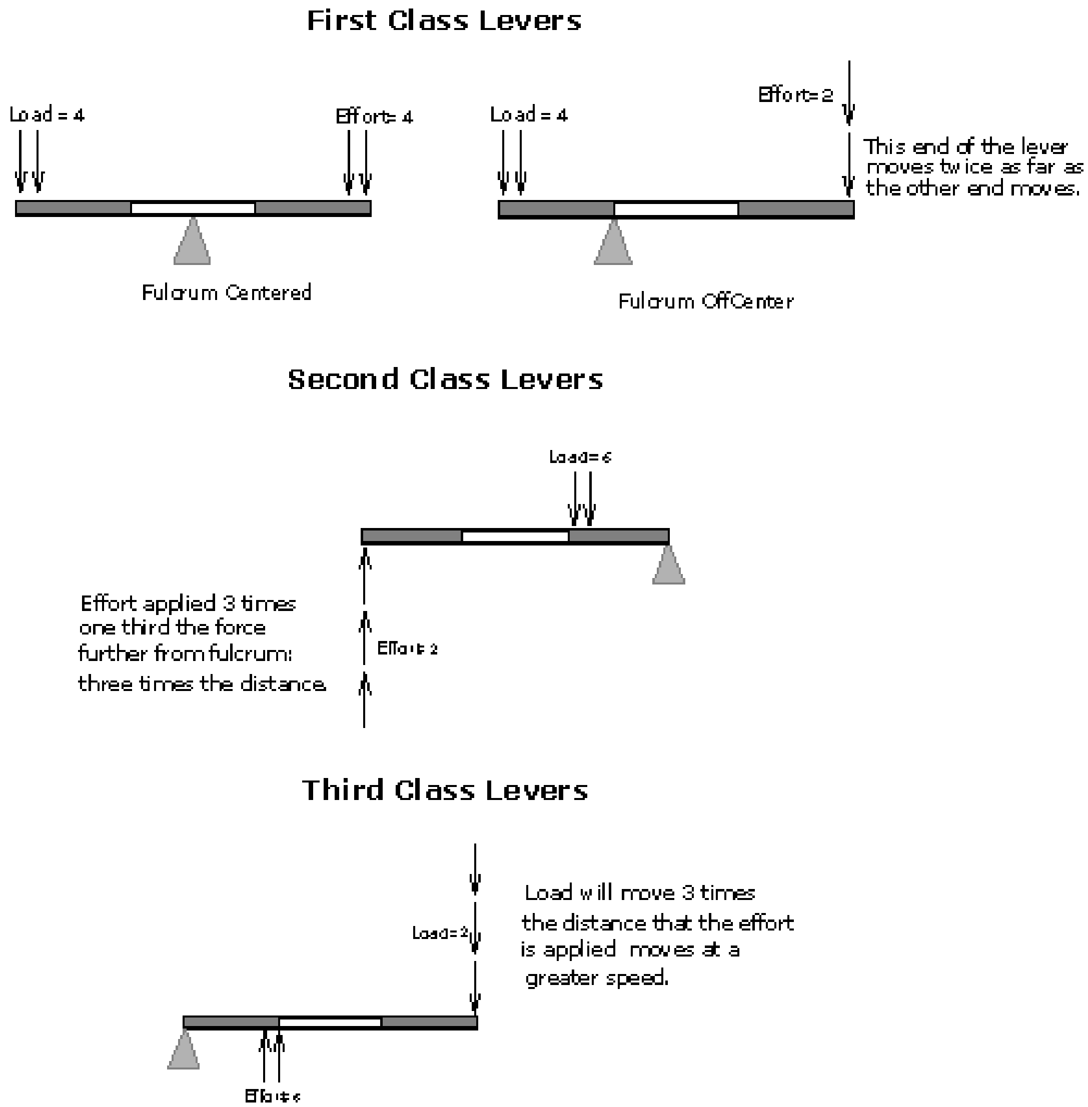

Figure 4: Classes of Levers

As shown in Figure 4, there are three different classes of levers defined by the relative positions of the fulcrum, effort, and load. A **first class lever** has the fulcrum positioned between the effort and the load. Examples of first class levers include: a balance, a crow bar, and scissors. In a **second class lever** the load is placed between the fulcrum and the effort. Examples of second class levers include: a wheelbarrow, a bottle opener, and a nutcracker. **Third class levers** place the effort between the fulcrum and the load.

Examples of a third class lever are a hammer, a fishing rod, and tweezers. Most machines that employ levers use a combination of several levers, often of different classes.

The Wheel and Axle

Both levers and the inclined plane lower the force required for a task at the price of having to apply that force over a longer distance. With wheels and axles the same is true: a poweful force and movement of the axle is converted to a greater movement, but less force, at the circumference of the wheel. In a circular geometry, **torque** is a more useful concept than force and distance. You can learn more about torque here[6]. The **wheel and axle** can be thought of as simply a circular lever, as shown in Figure 5. Many common items rely on the wheel and axle such as the screwdriver, the steering wheel, the wrench, and the faucet.

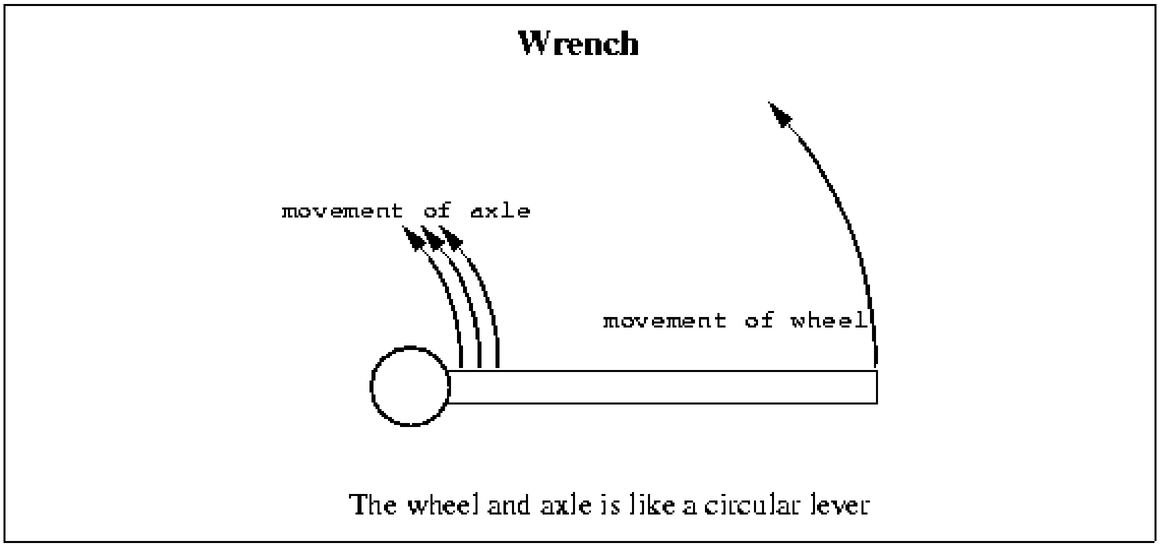

Figure 5: The Wheel and Axle

Gears and Belts

A wheel and axle assembly becomes especially useful when combined with gears and belts. **Gears** can be used to change the direction or speed of movement, but changing the speed of rotation inversely affects the force transmitted. A small gear meshed with a larger gear will turn faster, but with less force. There are four basic types of gears: spur gears, rack and pinion gears, bevel gears, and worm gears. **Spur gears** are probably the type of gear that most people picture when they hear the word. The two wheels are in the same plane (the axles are parallel). With **rack and pinion gears** there is one wheel and one rack, a flat toothed bar that converts the rotary motion into linear motion. **Bevel gears** are also known as pinion and crown or pinion and ring gears. In bevel gears, two wheels intermesh at an angle changing the direction of rotation (the axles are not parallel); the speed and force may also be modified, if desired. **Worm gears** involve one wheel gear (a pinion) and one shaft with a screw thread wrapped around it. Worm gears change

---

[6] "Introduction" <http://cnx.org/content/m11106/latest/>

the direction of motion as well as the speed and force. **Belts** work in the same manner as spur gears except that they do not change the direction of motion.

In both gears and belts, the speed and force is altered by the size of the two interacting wheels. In any pair, the bigger wheel always rotates more slowly, but with more force. On both the big and the small gear, the linear velocity at the point of contact for the wheels is equal. If it was unequal and one gear were spinning faster than the other at the point of contact then it would rip the teeth right off of the other gear. As the circumference of the larger gear is greater, a point on the outside of the larger gear must cover a greater distance than a point on the smaller gear to complete a revolution. Therefore the smaller gear must complete more revolutions than the larger gear in the same time span. (It's rotating more quickly.)

The force applied to the outer surface of each wheel must also be equal otherwise one of them would be accelerating more rapidly than the other, and again the teeth of the other wheel would break. However, the forces the forces applied to the axles are not equal. Returning to the concept of levers, we know that the distance from the fulcrum at which the force is applied effects the force applied at another point, and a wheel and axle works like a lever. Equal forces are being applied to each wheel, but on the larger wheel that force is being applied at a greater distance from the axle. Thus, for the larger wheel, the force on the axle is greater than the force on the axle of the smaller wheel.

Cams and Cranks

Both cams and cranks are useful when a repetitive motion is desired. **Cams** make rotary motion a little more interesting by essentially moving the axle off-center. Cams are often used in conjunction with a rod. One end of the rod is held flush against the cam by a spring. As the cam rotates the rod remains stationary until the "bump" of the cam pushes the rod away from the cam's axle. Cams can be used to create either a linear repetitive motion such as the one illustrated in Figure 6, or a repetitive rotational motion such as using a cam and a rubber band.

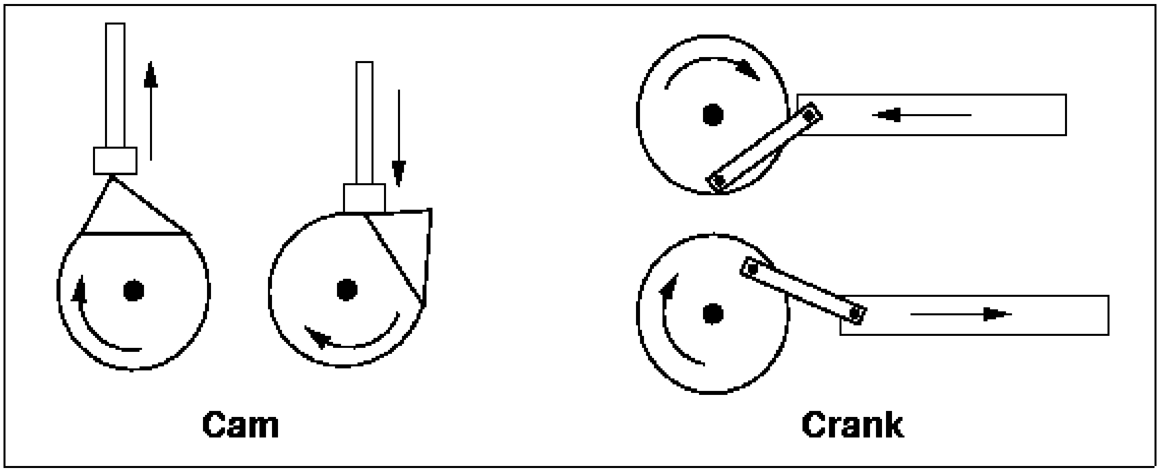

Figure 6: Cams and Cranks

Cranks convert rotary motion into a piston-like linear motion. The best examples of cranks in action are the drive mechanism for a steam locomotive and the automobile engine crankshaft. In a crank, the wheel rotates about a centered axle, while an arm is attached to the wheel with an off-centered peg. This arm is attached to a rod fixed in a linear path. A crank will cause the rod to move back and forth. If instead the rod is pushed back and forth, it will cause the crank to turn. On the other hand, cams can move their rods, but rods cannot move the cams.

Pulleys

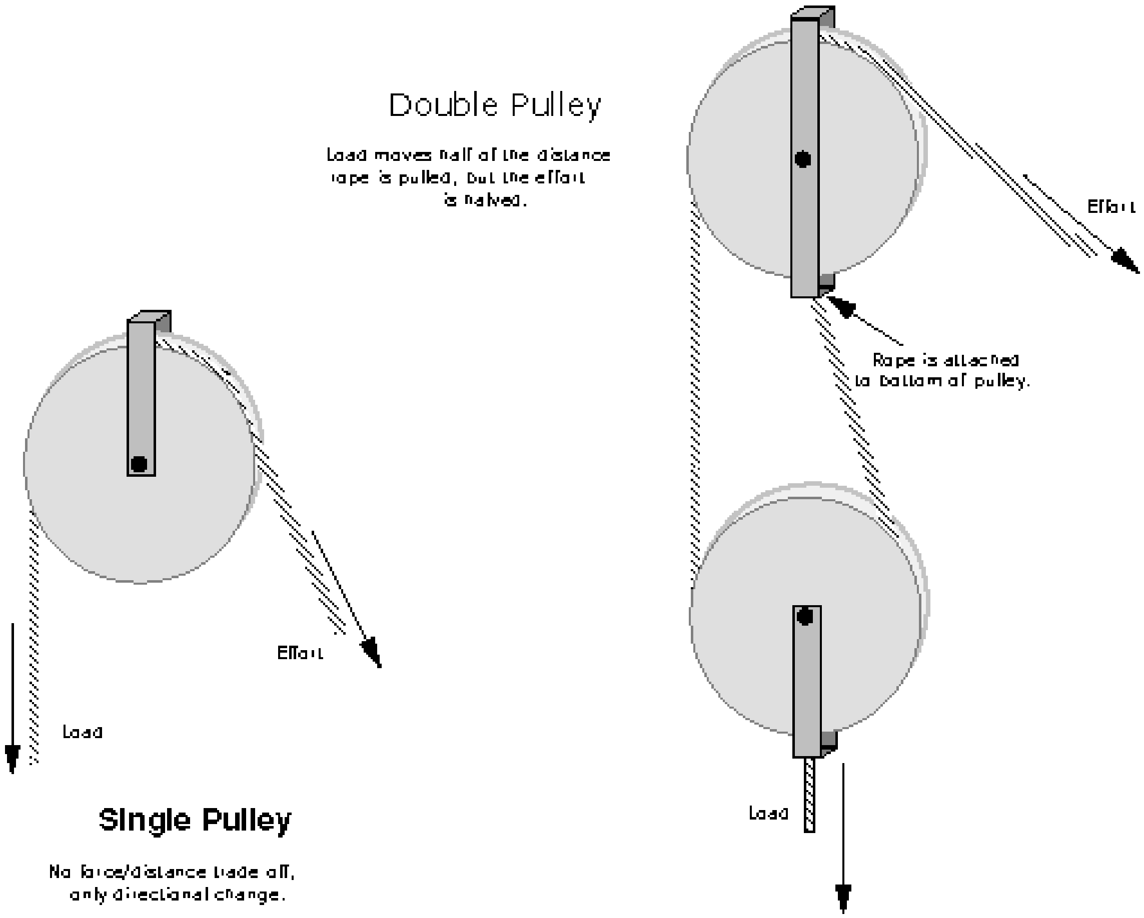

Figure 7: Pulleys

Pulleys can be used to simply change the direction of an applied force or to provide a force/distance tradeoff in addition to a directional change, as shown in Figure 7. Pulleys are very flexible because they use ropes or chain to transfer force rather than a rigid object such as a rod. Ropes can be routed through virtually any path. They are able to abruptly change directions in three-dimensions without consequence, except, of course, additional friction. Ropes can be wrapped around a motor's shaft and either wound up or let out as the motor turns.

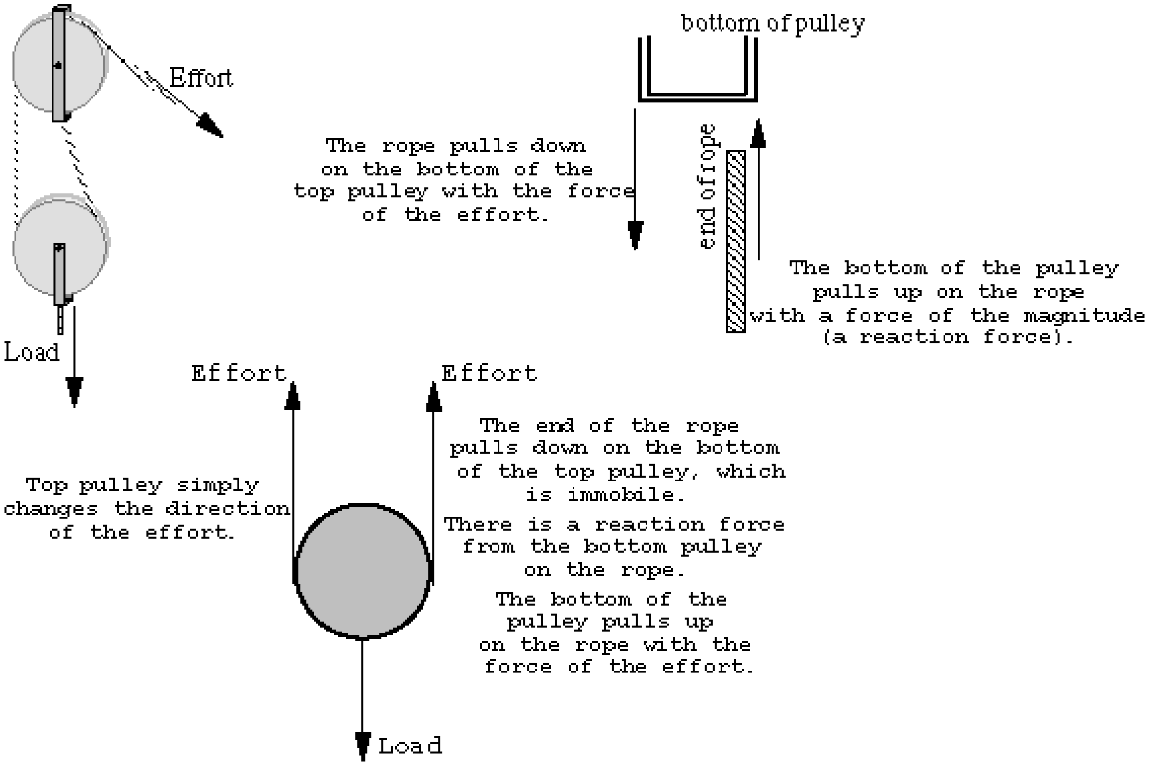

Figure 8: How Compound Pulleys Work

Figure 8 illustrates how a compound pulley 'trades' force for distance through an action/reaction force pair. In a double pulley, as the rope passes over the pulley the force is transmitted entirely but the direction has changed. The effort is now pulling up on the left side of the bottom pulley. Now, for a moment forget that the end of the rope is tied to the bottom of the top pulley. The mechanics are the same if the rope is fixed to the ceiling. The important thing is that the end of the rope is immobile. The effort is once again transmitted entirely as the rope passes over the bottom pulley and there is a direction change. The end of the rope is attached to the ceiling so the rope is pulling down on the ceiling with the force of the effort (and half of the force of the load). We assume that the ceiling holds up, so this must mean that there is a force balancing out this downward force. The ceiling pulls up on the rope as a reaction force. This upward force is equal to the effort and now there is an upward force on the right side of the bottom pulley. From the perspective of a free-body diagram the compound pulley system could be replaced by tying two ropes to the load and pulling up on each with a force equal to the effort.

The disadvantages of pulleys, in contrast to machines that use rigid objects to transfer force, are slipping and stretching. A rope will permanently stretch under tension, which may affect the future performance of a device. If a line becomes slack, then the operation of a machine may change entirely. Also, ropes will slip and stick along pulley wheels just like belts. One solution to the problems associated with rope is to use chain. Chain is pliable like rope, and is able to transfer force through many direction changes, but the chain links are inflexible in tension, so that the chain will not stretch. Chains may also be made to fit on gears so that slipping is avoided.

Springs[7]

A favorite device for storing potential energy is the **spring**. Everything from clocks to catapults make use of springs. One common use for springs is to return something to its original position. Another interesting application is the measurement (and creation of) force, such as springs in a scale. The third use is to store energy. All springs perform all three functions all of the time, but specific devices are built to exploit certain functions of the spring.

There are many physical forms of springs, created for specific purposes. The most basic form is the bending bar, shown in Figure 1. The familiar coil spring is just a bar spring in a different form. A rubber band is also a spring that makes use of the elastic properties of polymer materials.

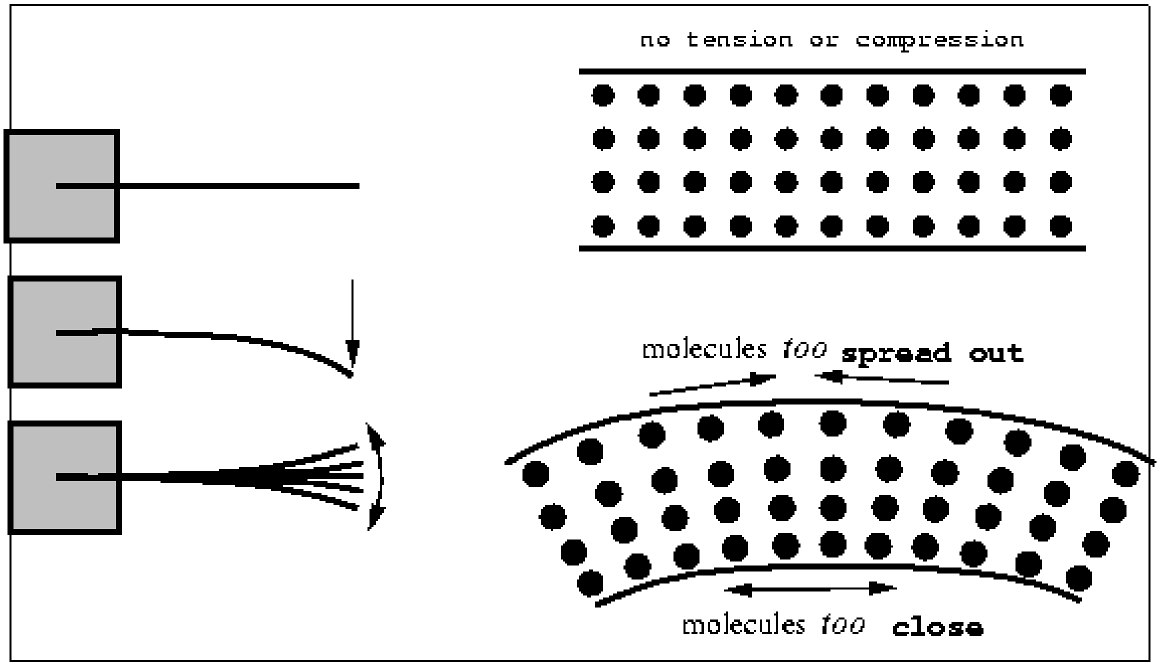

Figure 1: Bar Spring

To understand how a spring works, one must zoom in to the microscopic level where molecules interact.

---

[7] This content is available online at <http://cnx.org/content/m13545/1.1/>.

Molecules are held together in rigid bodies because of electromagnetic forces. Some of these forces are repulsive, and some of them are attractive. Normally they balance out so that the molecules are evenly spaced within an object; however, by bending a bar, some molecules are forced farther apart and others are shoved closer together, as in Figure 1. Where the molecules have been spread out, the attractive forces strive to return the original spacing. Where molecules have been forced together, the repulsive forces work to return the object to the original shape. A coil spring works in more or less the same way as a bar spring; when its shape is deformed, molecular forces act to return it to its original shape.

Rubber Bands

A rubber band is slightly more versatile than a metal spring because of its flexibility, just as pulleys are more versatile than their rigid cousin the lever. Using springs in a small robot might take a small amount of imagination, but rubber bands almost scream to be used. There are often several small tasks that a robot performs only once during a game. It would not make sense to devote an entire motor to such a task. It's not worth carrying around the extra weight if the task could be accomplished just as well with rubber bands. Examples include shooting a ball, pulling up a gate, or closing a jaw.

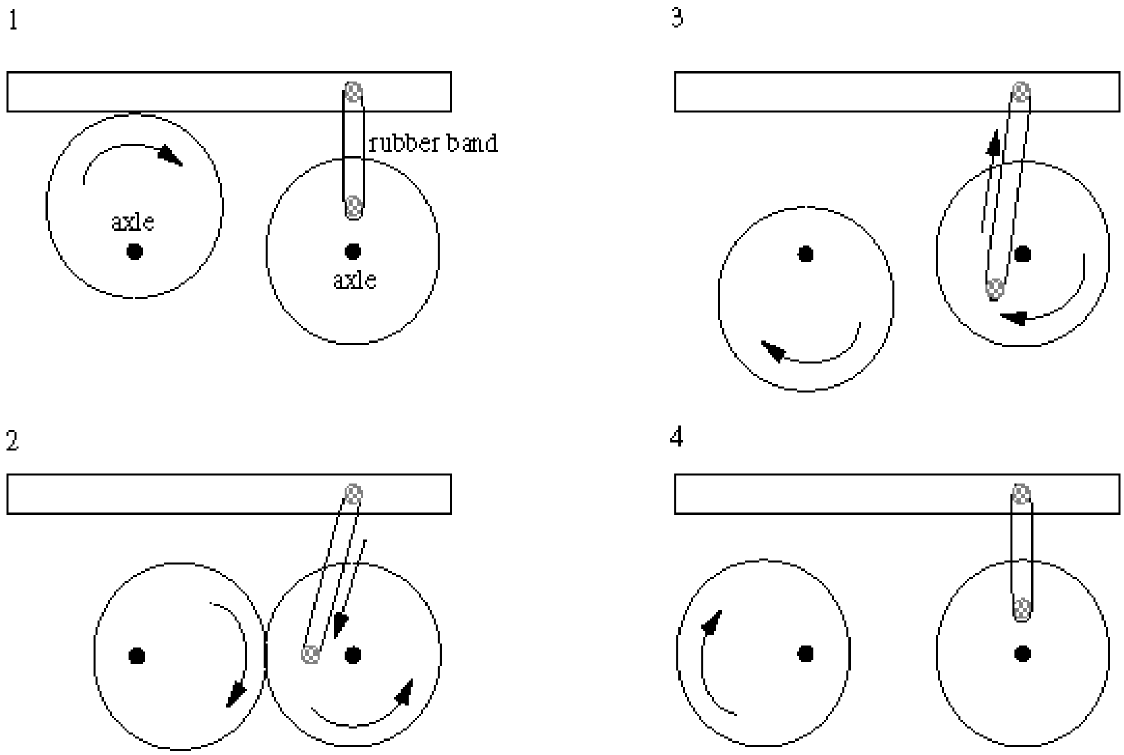

Figure 2: Using a Cam and a Rubber Band

Rubber bands also prove useful in the case of repetitive motions. Rather than turning a motor forward then backwards then forwards and so on, one could make use of a cam and a rubber band to allow the motor to always turn in one direction. Look at the assembly in Figure Figure 2 for an example.

Counterweights[8]

Counterweighting is a necessary evil in constructing even a simple robot. Small robots often carry a fairly massive battery, and its placement within the robot's structure is important. If the location of the robot's center of mass has not been well placed, there is a possibility of it overturning in response to sudden changes of speed, both stopping and starting. If the battery cannot be moved, weight may have to be added elsewhere to compensate. However, you may be able to use mechanical "outriders" to control tipping with much less added weight. Another use of counterweights is to keep a robot stable when a long arm extends, changing the center of mass. Examples of common **counterweights** are shown in Figure 1 .

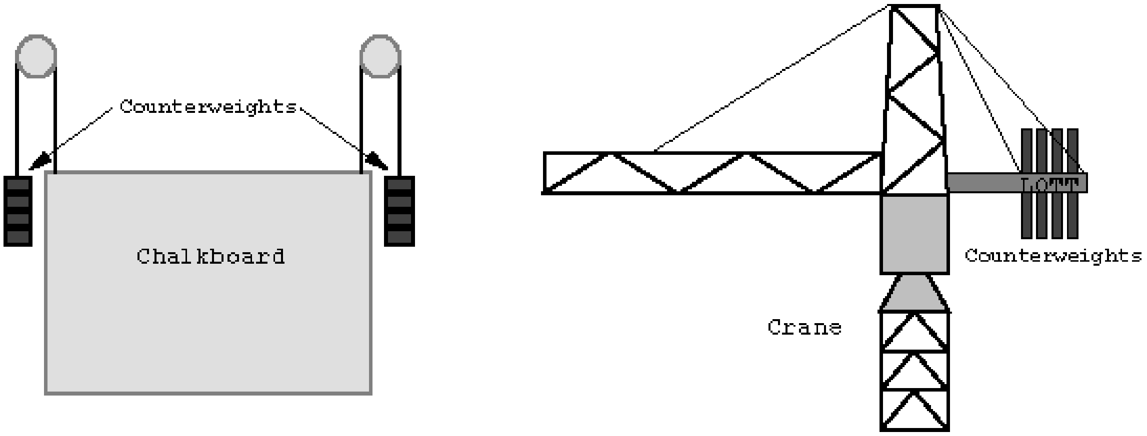

Figure 1: Some Common Counterweights

The use of counterweights might also prove useful to raise a bin carrying blocks. Rather than committing an entire motor to raising a bin, a set of counterweights, known to be heavier than the bin plus contents, could be suspended until the time when the bin should rise. Of course if a motor is used to release the the counterweights then no motors have been saved, but a much smaller motor and gear train could probably be used, or the motor could be used for more than one task if a mechanical transmission was employed. Another solution would be to use the high current LED outputs to operate a solenoid to release the counterweights. In addition, if the weight of a bin or an arm is balanced by a counterweight, it can be moved by a smaller, lighter motor, and there is less mechanical stress on the gears and mechanism.

---

[8] This content is available online at <http://cnx.org/content/m13546/1.1/>.

Chapter 1

Source Information

1.1 Rice ELEC 201 Course Notes Project[1]

The development the Rice University course ELEC 201, **Introduction to Engineering Design** included the compilation of an extensive set of class notes. The notes are not a text for the course, but rather a set of reference materials. We are currently in the process of transfering that information into Connexions modules and connecting related modules into courses.

Our notes were derived in part, with permission, from course notes developed by Fred Martin for the class 6.270 at the Massachusetts Institute of Technology. However, a considerable amount of new material has been added, and every subject area has undergone major revision for use at Rice with a class population from all majors. The printed version was a book over 400 pages long.

> NOTE: Fred Martin revised and extended his class notes to create the book *Robotic Explorations*, Prentice-Hall (2001) ISBN 0-13-089568-7. It is an excellent reference.

A number of Rice students have contributed to these notes. For the first version, Kymberly Maxham, our resident poet, labored to make the documentation accessible to individuals who are not science or engineering majors. In 1996, Patrick Frantz, Brendan Daly, and Jennifer Ngo, our new resident poet, all made major contributions. In the summer of 1997, Patrick Hearon and Alexis Beidenfeld put the notes online and made them more readable by humans, and in 1998, Anne Countiss made the on-line version of the book a reality.

The majority of the 1998 printed book is still available at the course web site, largely in its original form. However, web navigation within the book is, to put it kindly, inconvenient, primarily because some of the sections are quite long. We hope the Connexions version will solve many of those problems, and make the material more generally available. I thank the Connexions staff for their help in creating the initial modules.

The M.I.T. Department of Electrical Engineering and Computer Science and the M.I.T. Media Laboratory have agreed to unrestricted and free distribution of the robotics technology described in the course documentation for the 6.270 class. The Department of Electrical and Computer Engineering and the George R. Brown School of Engineering at Rice University have agreed to similar unrestricted distribution of the ELEC 201 class technology and material developed at Rice.

---

[1] This content is available online at <http://cnx.org/content/m13597/1.1/>.

Index of Keywords and Terms

Keywords are listed by the section with that keyword (page numbers are in parentheses). Keywords do not necessarily appear in the text of the page. They are merely associated with that section. *Ex.* apples, § 1.1 (1) **Terms** are referenced by the page they appear on. *Ex.* apples, 1

B Basic Mechanics, § (11)
Belts, 16
Bevel gears, 15

C Cams, 16
center of mass, § (23)
counterweight, § (23)
counterweights, 23
Cranks, 16

D Design, § 1.1(25)
dynamic friction, 7

E effort, 13
elasticity, § (19)
Engineering Design, § 1.1(25)

F first class lever, 14
Force, § (3)
free body diagram, 4
Friction, § (3), 7, 7
fulcrum, 13

G Gears, § (11), 15

I Inclined Plane, § (11), 11
Inertia, § (3), 3

L lever, § (9), § (11), 13
load, 13

M mechanics, § (1), 1

N normal force, 4

P Pulleys, 17

R rack and pinion gears, 15
rotational inertia, 3
rubber bands, § (19)

S Screw, § (11), 13
second class lever, 14
spring, 19
springs, § (19)
Spur gears, 15
Static friction, 7

T Third class levers, 14
torque, § (9), 9, 10, 15

W wheel and axle, 15
Worm gears, 15

Attributions

Collection: *Notes on Basic Mechanics for Rice ELEC 201*
Edited by: Jim Young
URL: http://cnx.org/content/col10357/1.1/

Module: "Introduction to Basic Mechanics"
By: Jim Young
URL: http://cnx.org/content/m13534/1.1/
Page: 1

Module: "Forces"
By: Jim Young
URL: http://cnx.org/content/m13535/1.2/
Pages: 3-7

Module: "Torque"
By: Jim Young
URL: http://cnx.org/content/m13544/1.1/
Pages: 9-10

Module: "Simple Machine Elements"
By: Jim Young
URL: http://cnx.org/content/m13594/1.1/
Pages: 11-18

Module: "Springs"
By: Jim Young
URL: http://cnx.org/content/m13545/1.1/
Pages: 19-21

Module: "Counterweights"
By: Jim Young
URL: http://cnx.org/content/m13546/1.1/
Page: 23

Module: "Rice ELEC 201 Course Notes Project"
By: Jim Young
URL: http://cnx.org/content/m13597/1.1/
Page: 25

Notes on Basic Mechanics for Rice ELEC 201

This "course" is the Basic Mechanics chapter of the Rice ELEC 201 design course notes. It presents information on basic mechanics that is useful for building small machines, like robots, and should be understandable by students from junior high upward.