The Fundamental Signal*

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Abstract

A description of the fundamental signal used in communications systems, the sinusoid, is presented as well as several of the properties of the sinusoid. The concept of modulation by a sinusoid is discussed.

1 The Sinusoid

The most ubiquitous and important signal in electrical engineering is the **sinusoid**.

Sine Definition

$$s(t) = A\cos(2\pi ft + \phi) \text{ or} A\cos(\omega t + \phi) \tag{1}$$

A is known as the sinusoid's **amplitude**, and determines the sinusoid's size. The amplitude conveys the sinusoid's physical units (volts, lumens, etc). The **frequency** f has units of Hz (Hertz) or s^{-1}, and determines how rapidly the sinusoid oscillates per unit time. The temporal variable t always has units of seconds, and thus the frequency determines how many oscillations/second the sinusoid has. AM radio stations have carrier frequencies of about 1 MHz (one mega-hertz or 10^6 Hz), while FM stations have carrier frequencies of about 100 MHz. Frequency can also be expressed by the symbol ω, which has units of radians/second. Clearly, $\omega = 2\pi f$. In communications, we most often express frequency in Hertz. Finally, ϕ is the **phase**, and determines the sine wave's behavior at the origin ($t = 0$). It has units of radians, but we can express it in degrees, realizing that in computations we must convert from degrees to radians. Note that if $\phi = -\frac{\pi}{2}$, the sinusoid corresponds to a sine function, having a zero value at the origin.

$$A\sin(2\pi ft + \phi) = A\cos\left(2\pi ft + \phi - \frac{\pi}{2}\right) \tag{2}$$

Thus, the only difference between a sine and cosine signal is the phase; we term either a sinusoid.

We can also define a discrete-time variant of the sinusoid: $A\cos(2\pi fn + \phi)$. Here, the independent variable is n and represents the integers. Frequency now has no dimensions, and takes on values between 0 and 1.

Exercise 1 ***(Solution on p. 3.)***

Show that $\cos(2\pi fn) = \cos(2\pi(f+1)n)$, which means that a sinusoid having a frequency larger than one corresponds to a sinusoid having a frequency less than one.

NOTE: Notice that we shall call either sinusoid an analog signal. Only when the discrete-time signal takes on a finite set of values can it be considered a digital signal.

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Exercise 2 ***(Solution on p. 3.)***

Can you think of a simple signal that has a finite number of values but is defined in continuous time? Such a signal is also an analog signal.

2 Communicating Information with Signals

The basic idea of communication engineering is to use a signal's parameters to represent either real numbers or other signals. The technical term is to **modulate** the **carrier** signal's parameters to transmit information from one place to another. To explore the notion of modulation, we can send a real number (today's temperature, for example) by changing a sinusoid's amplitude accordingly. If we wanted to send the daily temperature, we would keep the frequency constant (so the receiver would know what to expect) and change the amplitude at midnight. We could relate temperature to amplitude by the formula $A = A_0 (1 + kT)$, where A_0 and k are constants that the transmitter and receiver must **both** know.

If we had two numbers we wanted to send at the same time, we could modulate the sinusoid's frequency as well as its amplitude. This modulation scheme assumes we can estimate the sinusoid's amplitude and frequency; we shall learn that this is indeed possible.

Now suppose we have a sequence of parameters to send. We have exploited all of the sinusoid's two parameters. What we can do is modulate them for a limited time (say T seconds), and send two parameters every T. This simple notion corresponds to how a modem works. Here, typed characters are encoded into eight bits, and the individual bits are encoded into a sinusoid's amplitude and frequency. We'll learn how this is done in subsequent modules, and more importantly, we'll learn what the limits are on such digital communication schemes.

Solutions to Exercises in this Module

Solution to Exercise (p. 1)

As $\cos(\alpha+\beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$, $\cos(2\pi(f+1)n) = \cos(2\pi fn)\cos(2\pi n) - \sin(2\pi fn)\sin(2\pi n) = \cos(2\pi fn)$.

Solution to Exercise (p. 2)

A square wave takes on the values 1 and -1 alternately. See the plot in the module Elemental Signals[1].

[1] "Elemental Signals": Section Square Wave <http://cnx.org/content/m0004/latest/#squarewave>