

# IMPEDANCE EXAMPLE\*

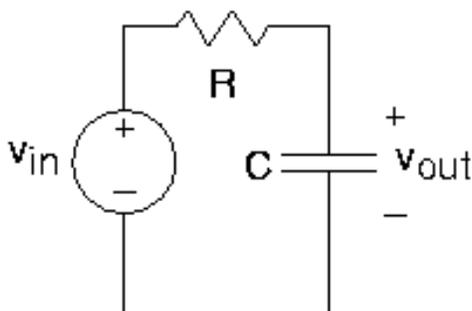
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## Abstract

Given an impedance Example to see how to use impedance.

To illustrate the impedance approach, we refer to the  $RC$  circuit (Figure 1) below, and we assume that  $v_{\text{in}} = V_{\text{in}}e^{i2\pi ft}$ .



**Figure 1:** A simple  $RC$  circuit.

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Using impedances, the complex amplitude of the output voltage  $V_{\text{out}}$  can be found using voltage divider:

$$V_{\text{out}} = \frac{Z_C}{Z_C + Z_R} V_{\text{in}} \quad (1)$$

$$V_{\text{out}} = \frac{\frac{1}{i2\pi fC}}{\frac{1}{i2\pi fC} + R} V_{\text{in}} \quad (2)$$

$$V_{\text{out}} = \frac{1}{i2\pi fRC + 1} V_{\text{in}} \quad (3)$$

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If we refer to the differential equation for this circuit (shown in Circuits with Capacitors and Inductors<sup>1</sup> to be  $RC \frac{dV_{\text{out}}}{dt} + V_{\text{out}} = V_{\text{in}}$ ), letting the output and input voltages be complex exponentials, we obtain the same relationship between their complex amplitudes. Thus, using impedances is equivalent to using the differential equation and solving it when the source is a complex exponential.

In fact, we can find the differential equation **directly** using impedances. If we cross-multiply the relation between input and output amplitudes,

$$V_{\text{out}} (i2\pi fRC + 1) = V_{\text{in}} \quad (4)$$

and then put the complex exponentials back in, we have

$$RCi2\pi fV_{\text{out}}e^{i2\pi ft} + V_{\text{out}}e^{i2\pi ft} = V_{\text{in}}e^{i2\pi ft} \quad (5)$$

In the process of defining impedances, note that the factor  $i2\pi f$  arises from the **derivative** of a complex exponential. We can reverse the impedance process, and revert back to the differential equation.

$$RC \frac{dV_{\text{out}}}{dt} + V_{\text{out}} = V_{\text{in}} \quad (6)$$

This is the same equation that was derived much more tediously in Circuits with Capacitors and Inductors<sup>2</sup>. Finding the differential equation relating output to input is far simpler when we use impedances than with any other technique.

#### Exercise 1

*(Solution on p. 3.)*

Suppose you had an expression where a complex amplitude was divided by  $i2\pi f$ . How did this division arise?

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<sup>1</sup>"Circuits with Capacitors and Inductors" <<http://cnx.org/content/m0023/latest/>>

<sup>2</sup>"Circuits with Capacitors and Inductors" <<http://cnx.org/content/m0023/latest/>>

## Solutions to Exercises in this Module

### Solution to Exercise (p. 2)

Division by  $i2\pi f$  arises from integrating a complex exponential. Consequently,

$$\frac{1}{i2\pi f} V \Leftrightarrow \int V e^{i2\pi f t} dt \quad (7)$$