Fourier Transform Example[*]

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Abstract

This module calculates the Fourier transform of the pulse signal.

Let's calculate the Fourier transform of the pulse signal[1], $p(t)$.

$$\begin{aligned} P(f) &= \int_{-\infty}^{\infty} p(t)\, e^{-(i2\pi f t)}\, dt \\ &= \int_0^{\Delta} e^{-(i2\pi f t)}\, dt \\ &= \frac{1}{-(i2\pi f)} \left(e^{-(i2\pi f \Delta)} - 1\right) \end{aligned} \tag{1}$$

$$P(f) = e^{-(i\pi f \Delta)} \frac{\sin(\pi f \Delta)}{\pi f} \tag{2}$$

Note how closely this result resembles the expression for Fourier series coefficients of the periodic pulse signal[2].

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[1] "Elemental Signals": Section Pulse <http://cnx.org/content/m0004/latest/#pulsedef>

[2] "The Periodic Pulse Signal", (2) <http://cnx.org/content/m0066/latest/#pulsespec>

Spectrum

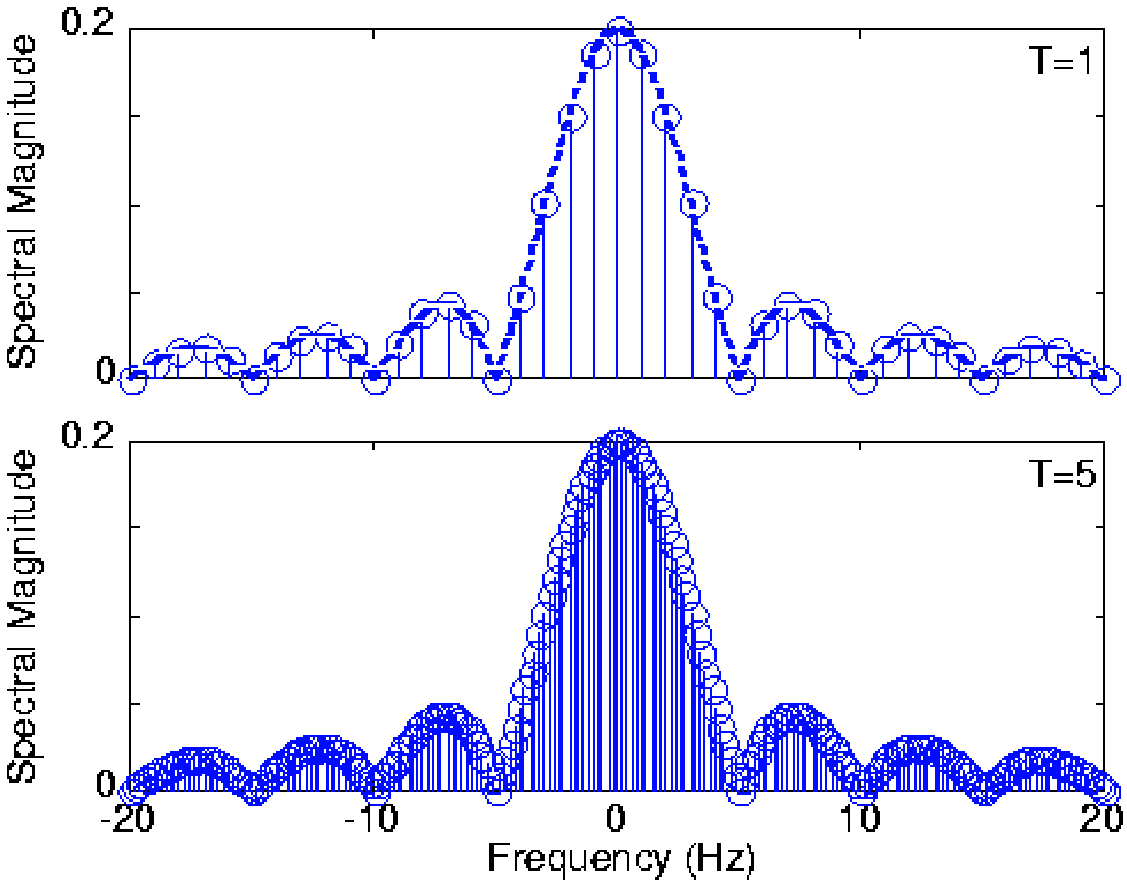

Figure 1: The upper plot shows the magnitude of the Fourier series spectrum for the case of $T = 1$ with the Fourier transform of $p(t)$ shown as a dashed line. For the bottom panel, we expanded the period to $T = 5$, keeping the pulse's duration fixed at 0.2, and computed its Fourier series coefficients.

Figure 1 (Spectrum) shows how increasing the period does indeed lead to a continuum of coefficients, and that the Fourier transform does correspond to what the continuum becomes. The quantity $\frac{\sin(t)}{t}$ has a special name, the **sinc** (pronounced "sink") function, and is denoted by $\text{sinc}(t)$. Thus, the magnitude of the pulse's Fourier transform equals $|\Delta \text{sinc}(\pi f \Delta)|$.