

# DISCRETE FOURIER TRANSFORM\*

Don Johnson

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## Abstract

The Fourier transform can be computed in discrete-time despite the complications caused by a finite signal and continuous frequency.

The discrete-time Fourier transform (and the continuous-time transform as well) can be evaluated when we have an analytic expression for the signal. Suppose we just have a signal, such as the speech signal used in the previous chapter. You might be curious; how did we compute a spectrogram such as the one shown in the speech signal example? The big difference between the continuous-time and discrete-time worlds is that we can **exactly** calculate spectra in discrete-time. For analog-signal spectra, use must build special devices, which turn out in most cases to consist of A/D converters and discrete-time computations. Certainly discrete-time spectral analysis is more flexible than in continuous-time.

The formula for the DTFT is a sum, which conceptually can be easily computed save for two issues.

- **Signal duration.** The sum extends over the signal's duration, which must be finite to compute the signal's spectrum. It is exceedingly difficult to store an infinite-length signal in any case, so we'll assume that the signal extends over  $[0, N - 1]$ .
- **Continuous frequency.** Subtler than the signal duration issue is the fact that the frequency variable is continuous: It may only need to span one period, like  $[-\frac{1}{2}, \frac{1}{2}]$  or  $[0, 1]$ , but the DTFT formula as it stands requires evaluating the spectra at **all** frequencies within a period. Let's compute the spectrum at a few frequencies; the most obvious ones are the equally spaced ones  $\forall k, k \in \{k, \dots, K - 1\} : (f = \frac{k}{K})$ .

We thus define the **discrete Fourier transform** (DFT) to be

### Discrete Fourier transform

$$\forall k, k \in \{k, \dots, K - 1\} : \left( S(k) = \sum_{n=0}^{N-1} S(n) e^{(-i)2\pi n \frac{k}{K}} \right) \quad (1)$$

Here,  $S(k)$  is shorthand for  $S\left(e^{i2\pi \frac{k}{K}}\right)$ .

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