Matched Filters in the Frequency Domain*

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Abstract

An analysis of matched filters in the frequency domain.

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The time domain analysis and implementation of matched filters can be found in Matched Filters[1].

A frequency domain interpretation of matched filters is very useful

$$\mathrm{SNR}=\frac{\left(\int_{-\infty}^{\infty}s_m(\tau)h_m(T-\tau)d\tau\right)^2}{\frac{N_0}{2}\int_{-\infty}^{\infty}(|h_m(T-\tau)|)^2d\tau} \quad (1)$$

For the m-th filter, h_m can be expressed as

$$\begin{array}{rcl}\tilde{s_m}(T) & = & \int_{-\infty}^{\infty}s_m(\tau)h_m(T-\tau)d\tau\\ & = & \mathcal{F}^{-1}(H_m(f)S_m(f))\\ & = & \int_{-\infty}^{\infty}H_m(f)S_m(f)e^{i2\pi fT}df\end{array} \quad (2)$$

where the second equality is because $\tilde{s}_m$ is the filter output with input S_m and filter H_m and we can now define $\hat{H_m}(f)=\overline{H_m(f)}e^{-(i2\pi fT)}$, then

$$\tilde{s_m}(T)=<S_m(f),\hat{H_m}(f)> \quad (3)$$

The denominator

$$\int_{-\infty}^{\infty}(|h_m(T-\tau)|)^2d\tau=\int_{-\infty}^{\infty}(|h_m(\tau)|)^2d\tau \quad (4)$$

$$\begin{array}{rcl}h_m*h_m(0) & = & \int_{-\infty}^{\infty}(|H_m(f)|)^2df\\ & = & <H_m(f),H_m(f)>\end{array} \quad (5)$$

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[1]"Matched Filters" <http://cnx.org/content/m10101/latest/>

$$\begin{aligned} h_m * h_m(0) &= \int_{-\infty}^{\infty} H_m(f)\, e^{i2\pi fT} \overline{H_m(f)} e^{-(i2\pi fT)} df \\ &= <\hat{H_m}(f), \hat{H_m}(f)> \end{aligned} \quad (6)$$

Therefore,

$$\text{SNR} = \frac{\left(< S_m(f), \hat{H_m}(f) >\right)^2}{\frac{N_0}{2} <\hat{H_m}(f), \hat{H_m}(f)>} \leq \frac{2}{N_0} < S_m(f), S_m(f) > \quad (7)$$

with equality when

$$\hat{H_m}(f) = \alpha S_m(f) \quad (8)$$

or

Matched Filter in the frequency domain

$$H_m(f) = \overline{S_m(f)} e^{-(i2\pi fT)} \quad (9)$$

Matched Filter

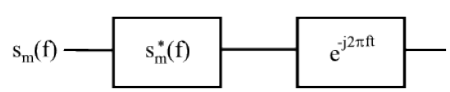

Figure 1

$$\begin{aligned} \tilde{s_m}(t) &= \mathcal{F}^{-1}\left(s_m(f)\, \overline{s_m(f)}\right) \\ &= \int_{-\infty}^{\infty} (|s_m(f)|)^2 e^{i2\pi ft} df \\ &= \int_{-\infty}^{\infty} (|s_m(f)|)^2 \cos(2\pi ft)\, df \end{aligned} \quad (10)$$

where $\mathcal{F}^{-1}$ is the inverse Fourier Transform operator.