# Performance Analysis of Antipodal Binary signals with Correlation*

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**Abstract**

Bit-error analysis for an antipodal signal set by using a correlator-type receiver.

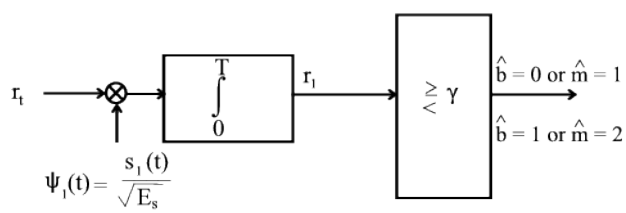


**Figure 1**

The bit-error probability for a correlation receiver with an antipodal signal set (Figure 1) can be found as follows:

$$\begin{aligned} P_e &= Pr\left[\hat{m} \neq m\right] \\ &= Pr\left[\hat{b} \neq b\right] \\ &= \pi_0 Pr\left[r_1 < \gamma|_{m=1}\right] + \pi_1 Pr\left[r_1 \geq \gamma|_{m=2}\right] \\ &= \pi_0 \int_{-\infty}^{\gamma} f_{r_1,s_1(t)}\left(r\right) dr + \pi_1 \int_{\gamma}^{\infty} f_{r_1,s_2(t)}\left(r\right) dr \end{aligned} \tag{1}$$

if $\pi_0 = \pi_1 = 1/2$, then the optimum threshold is $\gamma = 0$.

$$f_{r_1|s_1(t)}\left(r\right) = \mathscr{N}\left(\sqrt{E_s}, \frac{N_0}{2}\right) \tag{2}$$

$$f_{r_1|s_2(t)}\left(r\right) = \mathscr{N}\left(-\sqrt{E_s}, \frac{N_0}{2}\right) \tag{3}$$

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If the two symbols are equally likely to be transmitted then $\pi_0 = \pi_1 = 1/2$ and if the threshold is set to zero, then

$$P_e = 1/2 \int_{-\infty}^{0} \frac{1}{\sqrt{2\pi \frac{N_0}{2}}} e^{-\frac{(|r-\sqrt{E_s}|)^2}{N_0}} dr + 1/2 \int_{0}^{\infty} \frac{1}{\sqrt{2\pi \frac{N_0}{2}}} e^{-\frac{(|r+\sqrt{E_s}|)^2}{N_0}} dr \tag{4}$$

$$P_e = 1/2 \int_{-\infty}^{-\sqrt{\frac{2E_s}{N_0}}} \frac{1}{\sqrt{2\pi}} e^{\frac{-(|r'|)^2}{2}} dr\prime + 1/2 \int_{\sqrt{\frac{2E_s}{N_0}}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{\frac{-\left(|r''|\right)^2}{2}} dr'' \tag{5}$$

with $r' = \frac{r-\sqrt{E_s}}{\sqrt{\frac{N_0}{2}}}$ and $r'' = \frac{r+\sqrt{E_s}}{\sqrt{\frac{N_0}{2}}}$

$$\begin{aligned} P_e &= \tfrac{1}{2} Q\left(\sqrt{\tfrac{2E_s}{N_0}}\right) + \tfrac{1}{2} Q\left(\sqrt{\tfrac{2E_s}{N_0}}\right) \\ &= Q\left(\sqrt{\tfrac{2E_s}{N_0}}\right) \end{aligned} \tag{6}$$

where $Q(b) = \int_b^{\infty} \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}} dx$.

Note that

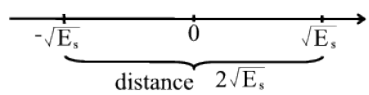


**Figure 2**

$$P_e = Q\left(\frac{d_{12}}{\sqrt{2N_0}}\right) \tag{7}$$

where $d_{12} = 2\sqrt{E_s} = (\| s_1 - s_2 \|)^2$ is the Euclidean distance between the two constellation points (Figure 2).

This is exactly the same bit-error probability as for the matched filter case.

A similar bit-error analysis for matched filters can be found here[1]. For the bit-error analysis for correlation receivers with an orthogonal signal set, refer here[2].

[1] "Performance Analysis of Binary Antipodal Signals with Matched Filters" <http://cnx.org/content/m10153/latest/>
[2] "Performance Analysis of Binary Orthogonal Signals with Correlation" <http://cnx.org/content/m10154/latest/>