

PROBABILITY EQUATIONS*

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Abstract

A collection of probability equations.

1 Probability equations

1.1 Simple Probability

$$Pr[x] = \frac{\text{Favorable Outcomes}}{\text{Possible Outcomes}} \quad (1)$$

Exercise 1

(Solution on p. 3.)

What is the probability that a card drawn at random from a deck of cards will be an ace?

1.2 Conditional Probability

A conditional probability is the probability of an event given that another event has occurred. For example, what is the probability that the total of two dice will be greater than 8 given that the first die is a 6? This can be computed by considering only outcomes for which the first die is a 6. Then, determine the proportion of these outcomes that total more than 8. All the possible outcomes for two dice are shown in the section on simple probability. There are 6 outcomes for which the first die is a 6, and of these, there are four that total more than 8 (6,3; 6,4; 6,5; 6,6). The probability of a total greater than 8 given that the first die is 6 is therefore $4/6 = 2/3$. More formally, this probability can be written as: $Pr[\text{total} > 8 \mid \text{Die 1} = 6] = 2/3$. In this equation, the expression to the left of the vertical bar represents the event and the expression to the right of the vertical bar represents the condition. Thus it would be read as "The probability that the total is greater than 8 given that Die 1 is 6 is $2/3$." In more abstract form, $Pr[A \mid B]$ is the probability of event A given that event B occurred.

1.3 Probability of A and B

Probability of A and B: Independent Variables

$$Pr[A \wedge B] = Pr[A] Pr[B] \quad (2)$$

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Exercise 2*(Solution on p. 3.)*

What is the probability that a fair coin will come up with heads twice in a row?

Probability of A and B: Dependent Variables

$$Pr[A \wedge B] = Pr[A] Pr[B | A] \quad (3)$$

Exercise 3*(Solution on p. 3.)*

If someone draws a card at random from a deck and then, without replacing the first card, draws a second card, what is the probability that both cards will be aces?

1.4 Probability of A or B**Probability of A or B: Mutually Exclusive Variables**

$$Pr[A \vee B] = Pr[A] + Pr[B] \quad (4)$$

Probability of A or B: Not Mutually Exclusive Variables

$$Pr[A \vee B] = Pr[A] + Pr[B] - Pr[A \wedge B] \quad (5)$$

1.5 Binomial Distribution

The binomial probability for obtaining r successes in N trials is:

$$Pr[r] = \frac{N!}{r!(N-r)!} \pi^r (1-\pi)^{N-r} \quad (6)$$

where $Pr[r]$ is the probability of exactly r successes, N is the number of events, and p is the probability of success on any one trial. This formula assumes that the events:

1. are dichotomous (fall into only two categories)
2. are mutually exclusive
3. are independent and
4. are randomly selected

Exercise 4*(Solution on p. 3.)*

Consider this simple application of the binomial distribution: What is the probability of obtaining exactly 3 heads if a fair coin is flipped 6 times?

Solutions to Exercises in this Module

Solution to Exercise 1 (p. 1)

Since of the 52 cards in the deck, 4 are aces, the probability is $4/52$.

Solution to Exercise 2 (p. 2)

Two events must occur: a head on the first toss and a head on the second toss. Since the probability of each event is $1/2$, the probability of both events is: $1/2 \times 1/2 = 1/4$.

Solution to Exercise 3 (p. 2)

Event A is that the first card is an ace. Since 4 of the 52 cards are aces, $Pr[A] = 4/52 = 1/13$. Given that the first card is an ace, what is the probability that the second card will be an ace as well? Of the 51 remaining cards, 3 are aces. Therefore, $Pr[B | A] = 3/51 = 1/17$ and the probability of A and B is: $1/13 \times 1/17 = 1/221$.

Solution to Exercise 4 (p. 2)

For this problem, $N = 6$, $r = 3$, and $\pi = 0.5$. Therefore,

$$Pr[3] = \frac{6!}{3!(6-3)!} 0.5^3 (1-0.5)^{6-3} = \frac{6 \times 5 \times 4 \times 3 \times 2}{(3 \times 2)(3 \times 2)} (0.125 \times 0.125) = 0.3125$$