Perfect Reconstruction FIR Filter Banks*

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Abstract

This module will drop the restrictive QMF conditions and focus on using FIR filters to achieve perfect reconstruction from filterbanks.

1 FIR Perfect-Reconstruction Conditions

The QMF[1] design choices prevented the design of a useful (*i.e.*, frequency selective) perfect-reconstruction (PR) FIR filterbank. This motivates us to re-examine PR filterbank design without the overly-restrictive QMF conditions. However, we will still require causal FIR filters with real coefficients.

Recalling that the two-channel filterbank[2] (Figure 1),

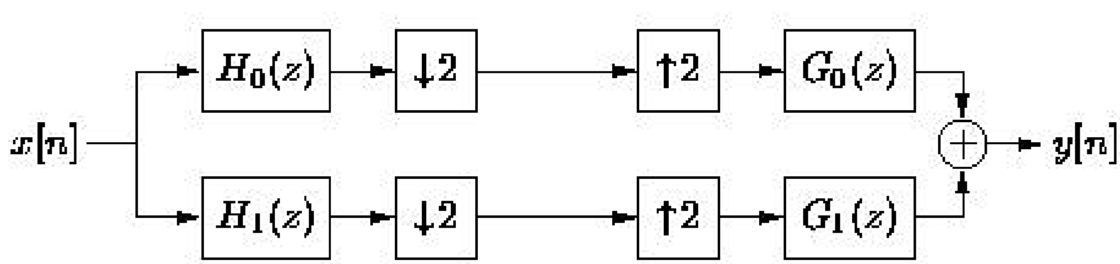

Figure 1

has the input/output relation:

$$Y(z) = \frac{1}{2} \begin{pmatrix} X(z) & X(-z) \end{pmatrix} \begin{pmatrix} H_0(z) & H_1(z) \\ H_0(-z) & H_1(-z) \end{pmatrix} \begin{pmatrix} G_0(z) \\ G_1(z) \end{pmatrix} \tag{1}$$

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[1]"Two-Branch Quadvalue Mirror Filterbank (QMF)" <http://cnx.org/content/m10426/latest/>

[2]"Aliasing-Cancellation Conditions of Filterbanks" <http://cnx.org/content/m10425/latest/>

we see that the delay-l perfect reconstruction requires

$$\begin{pmatrix} 2z^{-l} \\ 0 \end{pmatrix} = \begin{pmatrix} H_0(z) & H_1(z) \\ H_0(-z) & H_1(-z) \end{pmatrix} \begin{pmatrix} G_0(z) \\ G_1(z) \end{pmatrix} \quad (2)$$

where

$$H(z) = \begin{pmatrix} H_0(z) & H_1(z) \\ H_0(-z) & H_1(-z) \end{pmatrix}$$

or, equivalently, that

$$\begin{aligned} \begin{pmatrix} G_0(z) \\ G_1(z) \end{pmatrix} &= H^{-1}(z) \begin{pmatrix} 2z^{-l} \\ 0 \end{pmatrix} \\ &= \frac{1}{det(H(z))} \begin{pmatrix} H_1(-z) & -(H_1(z)) \\ -(H_0(-z)) & H_0(z) \end{pmatrix} \begin{pmatrix} 2z^{-l} \\ 0 \end{pmatrix} \\ &= \frac{2}{det(H(z))} \begin{pmatrix} z^{-l} H_1(-z) \\ -\left(z^{-l} H_0(-z)\right) \end{pmatrix} \end{aligned} \quad (3)$$

where

$$det(H(z)) = H_0(z) H_1(-z) - H_0(-z) H_1(z) \quad (4)$$

For FIR $G_0(z)$ and $G_1(z)$, we require [3] that

$$det(H(z)) = cz^{-k} \quad (5)$$

for $c \in \mathbb{R}$ and $k \in \mathbb{Z}$. Under this determinant condition, we find that

$$\begin{pmatrix} G_0(z) \\ G_1(z) \end{pmatrix} = \frac{2z^{-((l-k))}}{c} \begin{pmatrix} H_1(-z) \\ -(H_0(-z)) \end{pmatrix} \quad (6)$$

Assuming that $H_0(z)$ and $H_1(z)$ are causal with non-zero initial coefficient, we choose $k = l$ to keep $G_0(z)$ and $G_1(z)$ causal and free of unnecessary delay.

1.1 Summary of Two-Channel FIR-PR Conditions

Summarizing the two-channel FIR-PR conditions:

$$H_0(z) \ \wedge \ H_1(z) = \ \text{causal real} - \text{coefficient} \ \ \text{FIR}$$

$$\forall c, c \in \mathbb{R} \ \wedge \ l \in \mathbb{Z} : \left(det(H(z)) = cz^{-l}\right)$$

$$G_0(z) = \frac{2}{c} H_1(-z)$$

$$G_1(z) = \frac{-2}{c} H_0(-z)$$

[3]Since we cannot assume that FIR $H_0(z)$ and $H_1(z)$ share a common root.