

NORMED VECTOR SPACE*

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Abstract

This module introduces normed vector space.

Now we equip a vector space V with a notion of "size".

- A **norm** is a function ($\|\cdot\|: V \rightarrow \mathbb{R}$) such that the following properties hold ($\forall x, y, x \in V \wedge y \in V : (x \in V \wedge y \in V)$ and $\forall \alpha, \alpha \in \mathbb{F} : (\alpha \in \mathbb{F})$):
 - a. $\|x\| \geq 0$ with equality iff $x = 0$
 - b. $\|\alpha x\| = |\alpha| \cdot \|x\|$
 - c. $\|x + y\| \leq \|x\| + \|y\|$, (the **triangle inequality**).

In simple terms, the norm measures the size of a vector. Adding the norm operation to a vector space yields a **normed vector space**. Important examples include:

- a. $V = \mathbb{R}^N$, $\|(x_0, \dots, x_{N-1})^T\| := \sqrt{\sum_{i=0}^{N-1} x_i^2} := \sqrt{x^T x}$
- b. $V = \mathbb{C}^N$, $\|(x_0, \dots, x_{N-1})^T\| := \sqrt{\sum_{i=0}^{N-1} (|x_i|)^2} := \sqrt{x^H x}$
- c. $V = l_p$, $\| \{x[n]\} \| := \left(\sum_{n=-\infty}^{\infty} (|x[n]|)^p \right)^{\frac{1}{p}}$
- d. $V = \mathcal{L}_p$, $\|f(t)\| := \left(\int_{-\infty}^{\infty} (|f(t)|)^p dt \right)^{\frac{1}{p}}$

*Version 2.14: Oct 4, 2005 10:11 am +0000

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