The Complex Plane*

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1 Complex Plane

The **complex plane** provides a way to express complex numbers graphically. Any complex number can be expressed as a point on the complex plane. Before looking over the rest of this module, one should be very familiar with complex numbers. Please refer to the complex number[1] module for an explanation or review of these numbers.

Definition 1: Complex Plane
A two-dimensional graph where the horizontal axis maps the real part and the vertical axis maps the imaginary part of any complex number or function.

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[1]"Complex Numbers" <http://cnx.org/content/m0081/latest/>

Complex Plane

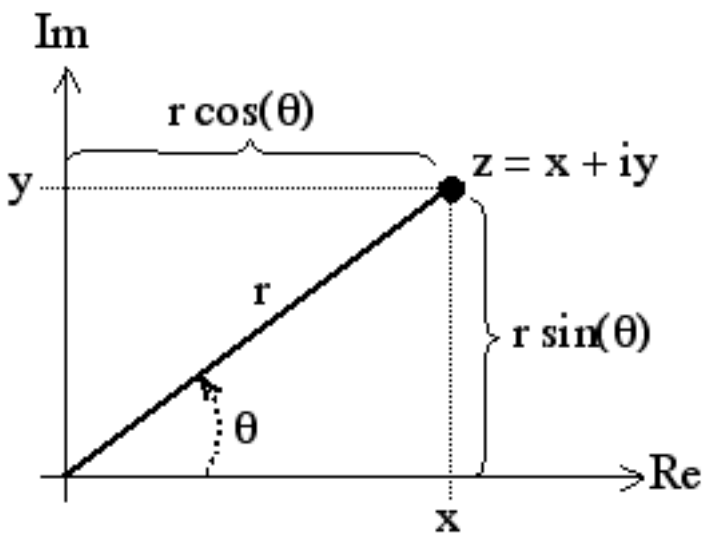

Figure 1: Plot of the complex number, z, as a point on the complex plane.

1.1 Rectangular Coordinates

Similar to the Cartesian plane, the complex plane allows one to plot ordered pairs of the form (a, b), where a and b are real numbers that describe a unique complex number through the following general form:

$$z = a + ib \tag{1}$$

This form is referred to as the **rectangular coordinate**.

1.2 Polar Form

The complex plane can also be used to plot complex numbers that are in **polar form**. Rather than using a and b, the polar coordinates use r and θ in their ordered pairs. The r is the distance from the origin to the complex number and θ is the angle of the complex number relative to the positive, real axis. Look at the figure above to see these variables displayed on the complex plane. The general form for polar numbers is as follows: $re^{i\theta}$

As a reminder, the following equations show the conversion between polar and rectangle coordinates:

$$r = \sqrt{a^2 + b^2} \tag{2}$$

$$\theta = \arctan\left(\frac{b}{a}\right) \tag{3}$$