Multirate Filtering: Theory Exercise*

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Abstract

You will work through an example problem that explores the effects of sample-rate compression and expansion on the spectrum of a signal.

1 Multirate Theory Exercise

Consider a sampled signal with the DTFT $X(\omega)$ shown in Figure 1.

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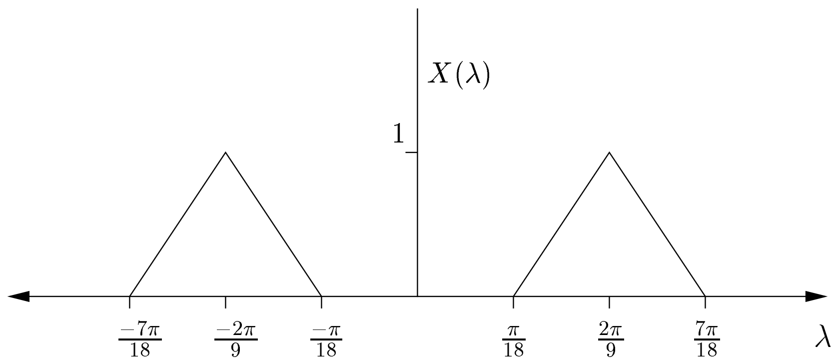

Figure 1: DTFT of the input signal.

Assuming $U = D = 3$, use the relations between the DTFT of a signal before and after sample-rate compression and expansion ((1) and (2)) to sketch the DTFT response of the signal as it passes through the multirate system of Figure 2 (without any filtering). Include both the intermediate response $W(\omega)$ and the final response $Y(\omega)$. It is important to be aware that the translation from digital frequency ω to analog frequency depends on the sampling rate. Therefore, the conversion is different for $X(\omega)$ and $W(\omega)$.

$$W(\omega) = \frac{1}{D} \sum_{k=0}^{D-1} X\left(\frac{\omega + 2\pi k}{D}\right) \tag{1}$$

$$Y(\omega) = W(U\omega) \tag{2}$$

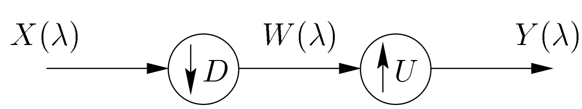

Figure 2: Multirate System