Introduction to Random Signals and Processes*

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Abstract

The module will introduce the concepts of a random signal and a random process.

Before now, you have probably dealt strictly with the theory behind signals and systems, as well as look at some the basic characteristics of signals[1] and systems[2]. In doing so you have developed an important foundation; however, most electrical engineers do not get to work in this type of fantasy world. In many cases the signals of interest are very complex due to the randomness of the world around them, which leaves them noisy and often corrupted. This often causes the information contained in the signal to be hidden and distorted. For this reason, it is important to understand these random signals and how to recover the necessary information.

1 Signals: Deterministic vs. Stochastic

For this study of signals and systems, we will divide signals into two groups: those that have a fixed behavior and those that change randomly. As most of you have probably already dealt with the first type, we will focus on introducing you to random signals. Also, note that we will be dealing strictly with discrete-time signals since they are the signals we deal with in DSP and most real-world computations, but these same ideas apply to continuous-time signals.

1.1 Deterministic Signals

Most introductions to signals and systems deal strictly with **deterministic signals**. Each value of these signals are fixed and can be determined by a mathematical expression, rule, or table. Because of this, future values of any deterministic signal can be calculated from past values. For this reason, these signals are relatively easy to analyze as they do not change, and we can make accurate assumptions about their past and future behavior.

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[1]"Signal Classifications and Properties" <http://cnx.org/content/m10057/latest/>
[2]"System Classifications and Properties" <http://cnx.org/content/m10084/latest/>

Deterministic Signal

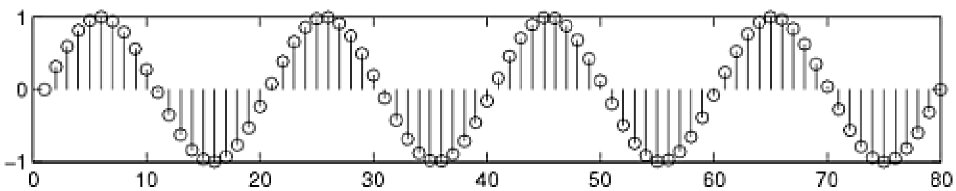

Figure 1: An example of a deterministic signal, the sine wave.

1.2 Stochastic Signals

Unlike deterministic signals, **stochastic signals**, or **random signals**, are not so nice. Random signals cannot be characterized by a simple, well-defined mathematical equation and their future values cannot be predicted. Rather, we must use probability and statistics to analyze their behavior. Also, because of their randomness, average values[3] from a collection of signals are usually studied rather than analyzing one individual signal.

Random Signal

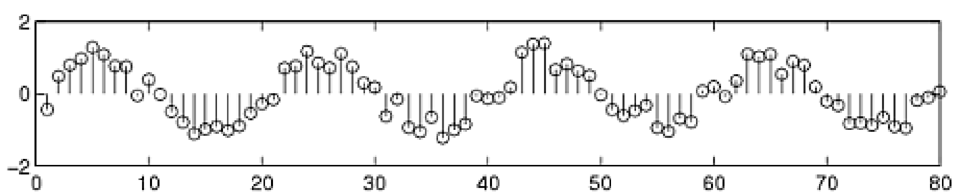

Figure 2: We have taken the above sine wave and added random noise to it to come up with a noisy, or random, signal. These are the types of signals that we wish to learn how to deal with so that we can recover the original sine wave.

2 Random Process

As mentioned above, in order to study random signals, we want to look at a collection of these signals rather than just one instance of that signal. This collection of signals is called a **random process**.

Definition 1: random process
A family or ensemble of signals that correspond to every possible outcome of a certain signal measurement. Each signal in this collection is referred to as a **realization** or **sample function** of the process.

[3] "Random Processes: Mean and Variance" <http://cnx.org/content/m10656/latest/>

Example

As an example of a random process, let us look at the Random Sinusoidal Process below. We use $f[n] = A\sin(\omega n + \phi)$ to represent the sinusoid with a given amplitude and phase. Note that the phase and amplitude of each sinusoid is based on a random number, thus making this a random process.

Random Sinusoidal Process

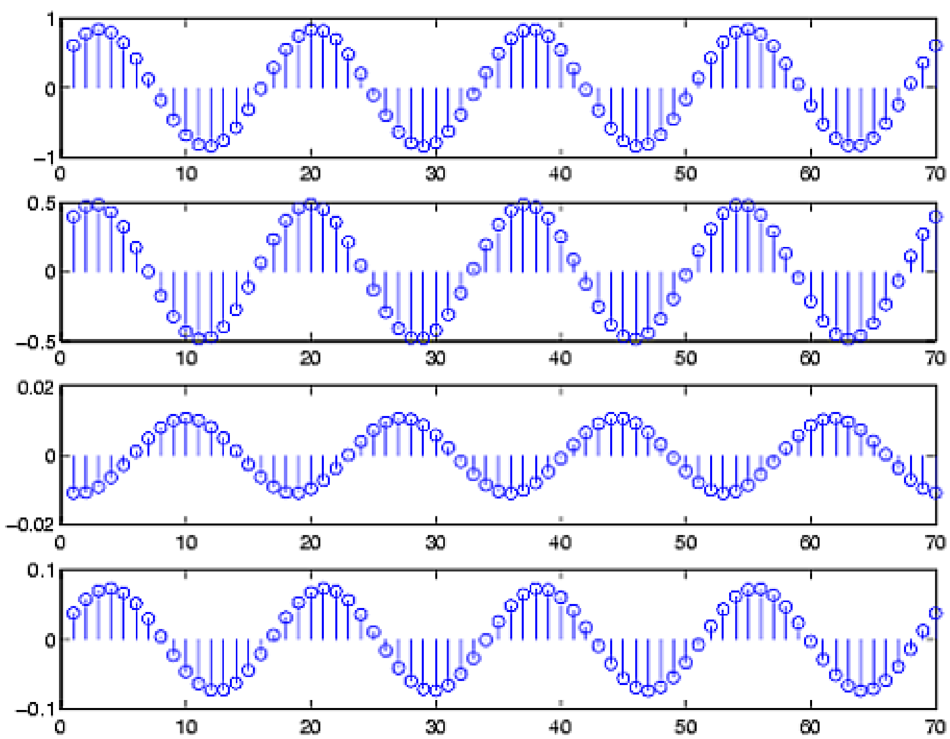

Figure 3: A random sinusoidal process, with the amplitude and phase being random numbers.

A random process is usually denoted by $X(t)$ or $X[n]$, with $x(t)$ or $x[n]$ used to represent an individual signal or waveform from this process.

In many notes and books, you might see the following notation and terms used to describe different types of random processes. For a **discrete random process**, sometimes just called a **random sequence**, t represents time that has a finite number of values. If t can take on any value of time, we have a **continuous random process**. Often times discrete and continuous refer to the amplitude of the process, and process or sequence refer to the nature of the time variable. For this study, we often just use **random process** to refer to a general collection of discrete-time signals, as seen above in Figure 3 (Random Sinusoidal Process).