Single Stub Matching*

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Abstract
Using the Smith Chart and a Single Stub to perform matching.

Often, there are reasons why using a discrete inductor or capacitor for matching is not such a good idea. At the high frequencies where matching is important, losses in both L or C mean that you don't get a good match, and most of the time (except for some air-dielectric adjustable capacitors) it is hard to get **just** the value you want.

There is another approach though. A shorted or open transmission line, when viewed at its input looks like a pure reactance or pure susceptance. With a short as a load, the reflection coefficient has unity magnitude $|\Gamma| = 1.0$ and so we move around the very outside of the Smith Chart (Figure 1: Input Impedance of a Shorted Line) as the length of the line increases or decreases, and $\frac{Z_{\mathrm{in}}}{Z_0}$ is purely imaginary. When we did the bilinear transformation from the $\frac{Z(s)}{Z_0}$ plane to the $r(s)$ plane, the imaginary axis transformed into the circle of diameter 2, which ended up being the outside circle which defined the Smith Chart.

Input Impedance of a Shorted Line

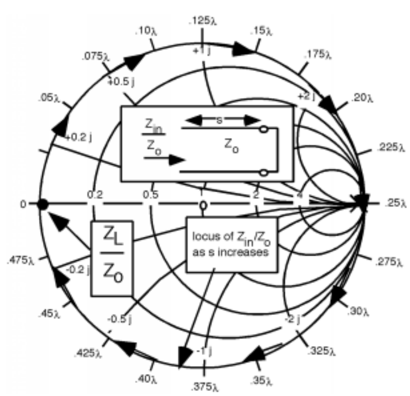

Figure 1

Another way to see this is to go back to this equation[1]. There we found:

$$Z(s) = Z_0 \frac{Z_L + iZ_0 \tan(\beta s)}{Z_0 + iZ_L \tan(\beta s)} \tag{1}$$

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[1] "Line Impedance", (3) <http://cnx.org/content/m1054/latest/#eqn3>

With $Z_L = 0$ this reduces to

$$Z(s) = iZ_0 \tan(\beta s) \qquad (2)$$

Which, of course for various values of s, can take on any value from $+i\infty$ to $-(i\infty)$. We don't have to go to Radio Shack©and buy a bunch of different inductor and capacitors. We can just get some transmission line and short it at various places!

Thus, instead of a discrete component, we can use a section of shorted (or open) transmission line instead Figure 2 (A Shortened Stub). These matching lines are called **matching stubs**. One of the major advantages here is that with a line which has an adjustable short on the end of it, we can get any reactance we need, simply by adjusting the length of the stub. How this all works will become obvious after we take a look at an example.

A Shortened Stub

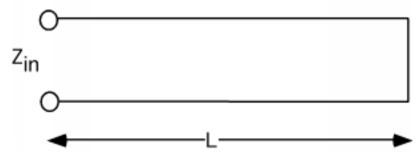

Figure 2

Let's do one. In Figure 3 (Another Load) we can see that, $\frac{Z_L}{Z_0} = 0.2 + 0.5i$, so we mark a point "A" on the Smith Chart. Since we will want to put the tuning or matching stub in shunt across the line, the first thing we will do is convert $\frac{Z_L}{Z_0}$ into a normalized admittance $\frac{Y_L}{Y_0}$ by going 180° around the Smith Chart (Figure 4: Converting to Normalized Admittance) to point "B", where $\left(\frac{Y_L}{Y_0} \approx 0.7 - 1.7i\right)$. Now we rotate around on the constant radius, $r(s)$ circle until we hit the matching circle at point "C". This is shown in Figure 5 (Moving to the Matching Circle). At "C", $\frac{Y_S}{Y_0} = 1.0 + 2.0i$. Using a "real" Smith Chart, I get that the distance of rotation is about 0.36λ. Remember, all the way around is $\frac{\lambda}{2}$, so you can very often "eyeball" about how far you have to go, and doing so is a good check on making a stupid math error. If the distance doesn't look right on the Smith Chart, you probably made a mistake!

Another Load

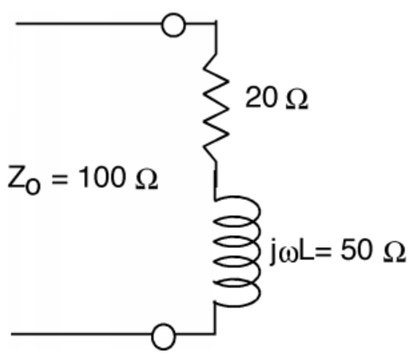

Figure 3

Converting to Normalized Admittance

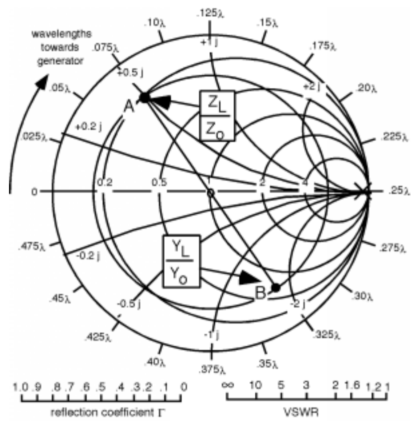

Figure 4: Converting to $\frac{Y_L}{Y_0}$

Moving to the Matching Circle

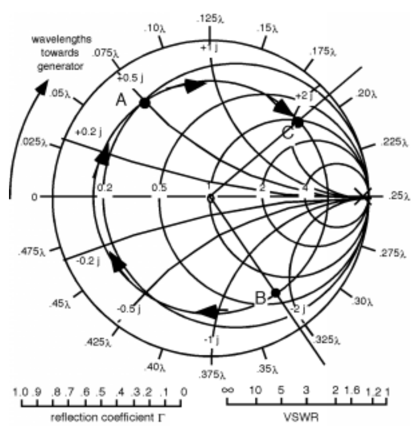

Figure 5

OK, at this point, the real part of the admittance is unity, so all we have to do is add a stub to cancel out the imaginary part. As mentioned above, the stubs often come with adjustable, or "sliding short" so we can make them whatever length we want Figure 6 (Matching with a Shortened Stub).

Matching with a Shortened Stub

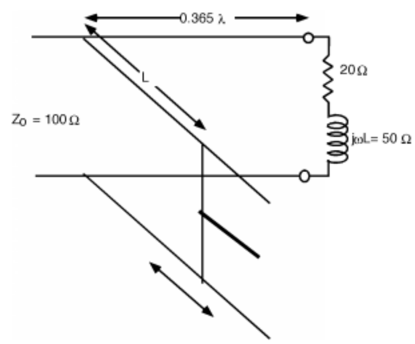

Figure 6

Our task now, is to decide how much to push or pull on the sliding handle on the stub, to get the reactance we want. The hint on what we should do is in Figure 1 (Input Impedance of a Shorted Line). The end of the stub is a short circuit. What is the admittance of a short circuit? Answer: ∞, $i\infty$! Where is this on the Smith Chart? Answer: on the outside, on the right hand side on the real axis. Now, if we start at a short, and start to make the line longer than $s = 0$, what happens to $\frac{Y(s)}{Y_0}$? It moves around on the outside of the Smith Chart. What we need to do is move away from the short until we get $\frac{Y(s)}{Y_0} = -(i2.0)$ and we will know how long the shorted tuning stub should be Figure 7 (Finding the Stub length). In going from "A" to "B" we traverse a distance of about 0.07λ and so that is where we should set the position of the sliding short on the stub Figure 8 (The Matched Line).

Finding the Stub length

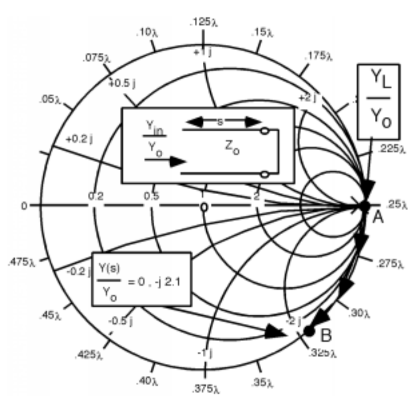

Figure 7

The Matched Line

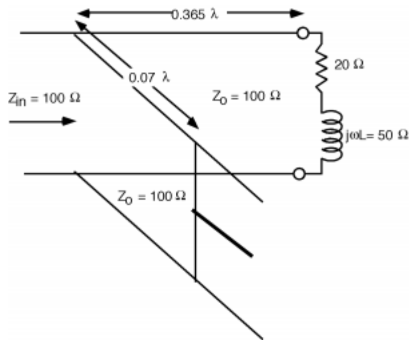

Figure 8

We sometimes think of the action of the tuning stub as allowing us to move in along the $\Re\left(\frac{Y(s)}{Y_0}\right)$ to get to the center of the Smith Chart, or to a match Figure 9 (Moving With a Stub). We are not in this case, physically moving down the line. Rather we are moving along a **contour of constant real part** because all the stub can do is change the imaginary part of the admittance, it can do nothing to the real part!

Moving With a Stub

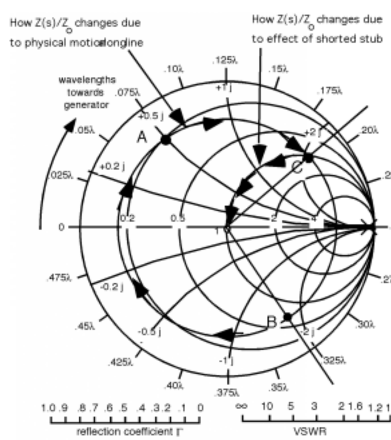

Figure 9: Moving along the $\Re\left(\frac{Y(s)}{Y_0}\right) = 1$ circle with a stub.