Crosscorrelation of Random Processes*

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Before diving into a more complex statistical analysis of random signals and processes[1], let us quickly review the idea of correlation[2]. Recall that the correlation of two signals or variables is the expected value of the product of those two variables. Since our main focus is to discover more about random processes, a collection of random signals, we will deal with two random processes in this discussion, where in this case we will deal with samples from two **different** random processes. We will analyze the expected value[3] of the product of these two variables and how they correlate to one another, where the argument to this correlation function will be the time difference. For the correlation of signals from the same random process, look at the autocorrelation function[4].

1 Crosscorrelation Function

When dealing with multiple random processes, it is also important to be able to describe the relationship, if any, between the processes. For example, this may occur if more than one random signal is applied to a system. In order to do this, we use the **crosscorrelation function**, where the variables are instances from two different wide sense stationary random processes.

Definition 1: Crosscorrelation
if two processes are wide sense stationary, the expected value of the product of a random variable from one random process with a time-shifted, random variable from a different random process

Looking at the generalized formula for the crosscorrelation, we will represent our two random processes by allowing $U = U(t)$ and $V = V(t-\tau)$. We will define the crosscorrelation function as

$$\begin{aligned} R_{uv}(t, t-\tau) &= E[UV] \\ &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} uvf(u,v)\,dvdu \end{aligned} \tag{1}$$

Just as the case with the autocorrelation function, if our input and output, denoted as $U(t)$ and $V(t)$, are at least jointly wide sense stationary, then the crosscorrelation does not depend on absolute time; it is just a function of the time difference. This means we can simplify our writing of the above function as

$$R_{uv}(\tau) = E[UV] \tag{2}$$

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[1]"Introduction to Random Signals and Processes" <http://cnx.org/content/m10649/latest/>
[2]"Correlation and Covariance of a Random Signal" <http://cnx.org/content/m10673/latest/>
[3]"Random Processes: Mean and Variance": Section Mean Value <http://cnx.org/content/m10656/latest/#sec1>
[4]"Autocorrelation of Random Processes" <http://cnx.org/content/m10676/latest/>

or if we deal with two real signal sequences, $x[n]$ and $y[n]$, then we arrive at a more commonly seen formula for the discrete crosscorrelation function. See the formula below and notice the similarities between it and the convolution[5] of two signals:

$$\begin{aligned} R_{xy}(n, n-m) &= R_{xy}(m) \\ &= \sum_{n=-\infty}^{\infty} x[n]\, y[n-m] \end{aligned} \tag{3}$$

1.1 Properties of Crosscorrelation

Below we will look at several properties of the crosscorrelation function that hold for two **wide sense stationary (WSS)** random processes.

- Crosscorrelation is **not** an even function; however, it does have a unique symmetry property:

$$R_{xy}(-\tau) = R_{yx}(\tau) \tag{4}$$

- The maximum value of the crosscorrelation is not always when the shift equals zero; however, we can prove the following property revealing to us what value the maximum cannot exceed.

$$|R_{xy}(\tau)| \leq \sqrt{R_{xx}(0)\, R_{yy}(0)} \tag{5}$$

- When two random processes are statistically independent then we have

$$R_{xy}(\tau) = R_{yx}(\tau) \tag{6}$$

2 Examples

Exercise 1 ***(Solution on p. 3.)***

Let us begin by looking at a simple example showing the relationship between two sequences. Using (3), find the crosscorrelation of the sequences

$$x[n] = \{\ldots, 0, 0, 2, -3, 6, 1, 3, 0, 0, \ldots\}$$

$$y[n] = \{\ldots, 0, 0, 1, -2, 4, 1, -3, 0, 0, \ldots\}$$

for each of the following possible time shifts: $m = \{0, 3, -1\}$.

[5] "Discrete Time Convolution" <http://cnx.org/content/m10087/latest/>

Solutions to Exercises in this Module

Solution to Exercise (p. 2)

1. For $m = 0$, we should begin by finding the product sequence $s[n] = x[n]\,y[n]$. Doing this we get the following sequence:
$$s[n] = \{\ldots, 0, 0, 2, 6, 24, 1, -9, 0, 0, \ldots\}$$
and so from the sum in our crosscorrelation function we arrive at the answer of
$$R_{xy}(0) = 22$$
2. For $m = 3$, we will approach it the same was we did above; however, we will now shift $y[n]$ to the right. Then we can find the product sequence $s[n] = x[n]\,y[n-3]$, which yields
$$s[n] = \{\ldots, 0, 0, 0, 0, 0, 1, -6, 0, 0, \ldots\}$$
and from the crosscorrelation function we arrive at the answer of
$$R_{xy}(3) = -6$$
3. For $m = -1$, we will again take the same approach; however, we will now shift $y[n]$ to the left. Then we can find the product sequence $s[n] = x[n]\,y[n+1]$, which yields
$$s[n] = \{\ldots, 0, 0, -4, -12, 6, -3, 0, 0, 0, \ldots\}$$
and from the crosscorrelation function we arrive at the answer of
$$R_{xy}(-1) = -13$$