

# EIGEN-STUFF IN A NUTSHELL\*

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## Abstract

This module provides a brief overview and review of the importance of eigenvectors and eigenvalues in analyzing and understanding LTI systems.

## 1 A Matrix and its Eigenvector

The reason we are stressing eigenvectors and their importance is because the action of a matrix  $A$  on one of its eigenvectors  $v$  is

1. extremely easy (and fast) to calculate

$$Av = \lambda v \quad (1)$$

just **multiply**  $v$  by  $\lambda$ .

2. easy to interpret:  $A$  just **scales**  $v$ , keeping its direction constant and only altering the vector's length.

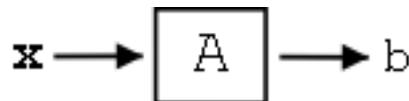
If only every vector were an eigenvector of  $A$ ...

## 2 Using Eigenvectors' Span

Of course, not every vector can be ... BUT ... For certain matrices (including ones with distinct eigenvalues,  $\lambda$ 's), their eigenvectors span  $\mathbb{C}^n$ , meaning that for **any**  $x \in \mathbb{C}^n$ , we can find  $\{\alpha_1, \alpha_2, \alpha_n\} \in \mathbb{C}$  such that:

$$x = \alpha_1 v_1 + \alpha_2 v_2 + \cdots + \alpha_n v_n \quad (2)$$

Given (2), we can rewrite  $Ax = b$ . This equation is modeled in our LTI system pictured below:



**Figure 1:** LTI System.

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$$x = \sum_i \alpha_i v_i$$

$$b = \sum_i \alpha_i \lambda_i v_i$$

The LTI system above represents our (1). Below is an illustration of the steps taken to go from  $x$  to  $b$ .

$$x \rightarrow (\alpha = V^{-1}x) \rightarrow (\Lambda V^{-1}x) \rightarrow (V\Lambda V^{-1}x = b)$$

where the three steps (arrows) in the above illustration represent the following three operations:

1. Transform  $x$  using  $V^{-1}$  - yields  $\alpha$
2. Action of  $A$  in new basis - a multiplication by  $\Lambda$
3. Translate back to old basis - inverse transform using a multiplication by  $V$ , which gives us  $b$