Percentiles*

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Abstract

This module gives the definition of percentile along with several examples

A test score in and of itself is usually difficult to interpret. For example, if you learned that your score on a measure of shyness were 35 out of a possible 50, you would have little idea how shy you are compared to other people. More relevant is the **percentage** of people with lower shyness scores than yours.

Definition 1: Percentile
The percentage with lower shyness scores than yours.
Example
If 65% of the scores were below yours, then your score would be the 65th percentile.

1 Two Simple Definitions of Percentile

There is no universally accepted definition of a percentile. Using the 65th percentile as an example, the 65th percentile can be defined as the lowest score that is greater than 65% of the scores. This is the way we defined it above and we will call this "Definition 1". The 65th percentile can also be defined as the smallest score that is greater than or equal to 65% of the scores. This we will call "Definition 2". Unfortunately, these two definitions can lead to dramatically different results, especially when there is relatively little data. Moreover, neither of these definitions is explicit about how to handle rounding. For instance, what score is required to be higher than 65% of the scores when the total number of scores is 50? This is tricky because 65% of 50 is 32.5. How do we find the lowest number that is higher than 32.5 of the scores? A third way to compute percentiles (presented below), is a weighted average of the percentiles computed according to the first two definitions. This third definition handles rounding more gracefully than the other two and has the advantage that it allows the median[1] (discussed later[2]) to be defined conveniently as the 50th percentile.

2 A Third Definition

Unless otherwise specified, when we refer to "percentile", we will be referring to this third definition of percentiles. Let's begin with an example.

Example 1
Consider the 25th percentile for the 8 numbers in the table (Table 1: Test Scores). Notice the numbers are given ranks ranging from 1 for the lowest number to 8 for the highest number.

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[1]http://psych.rice.edu/online_stat/glossary/median.html
[2]"Median and Mean" <http://cnx.org/content/m11165/latest/>

Test Scores

Number	Rank
3	1
5	2
7	3
8	4
9	5
11	6
13	7
15	8

Table 1

The first step is to compute the rank (R) of the 25th percentile. This is done using the following formula:

$$R = \frac{P}{100}(N+1)$$

where P is the desired percentile (25 in this case) and N is the number of numbers (8 in this case). Therefore,

$$R = \frac{25}{100}(8+1) = \frac{9}{4} = 2.25$$

If R were an integer, the $P$the percentile would be the number with rank R. When R is not an integer, we compute the Pth percentile by interpolation as follows:

1. Define IR as the integer portion of R (the number to the left of the decimal point). For this example, IR = 2
2. Define FR as the fractional portion of R. For this example, FR = 0.25
3. Find the scores with Rank I_R and with Rank $I_R + 1$ For this example, this means the score with Rank 2 and the score with Rank 3. The scores are 5 and 7.
4. Interpolate by multiplying the difference between the scores by F_R and add the result to the lower score. For these data, this is $0.25 \times (7-5) + 5 = 5.5$

Therefore, the 25th percentile is 5.5. If we had used the first definition (the smallest score greater than 25% of the scores) the 25th percentile would have been 7. If we had used the second definition (the smallest score greater than or equal to 25% of the scores) the 25th percentile would have been 5.

Example 2

For a second example, consider the 20 quiz scores in the table (Table 2: 20 Quiz Scores).

20 Quiz Scores

Score	Rank
4	1
4	2
5	3
5	4
5	5
5	6
6	7
6	8
6	9
7	10
7	11
7	12
8	13
8	14
9	15
9	16
9	17
10	18
10	19
10	20

Table 2

We will compute the 25th and the 85th percentiles. For the 25th,

$$R = \frac{25}{100}(20+1) = \frac{21}{4} = 5.25$$

$$\text{IR} = 5$$

$$\text{FR} = 0.25$$

Since the score with a rank of IR (which is 5) and the score with a rank of IR + 1 (which is 6) are both equal to 5, the 25th percentile is 5. In terms of the formula:

The 25th percentile equals

$$0.25 \times (5-5) + 5 = 5$$

For the 85th percentile,

$$R = \frac{85}{100}(20+1) = 17.85$$

$$\text{IR} = 17$$

$$\text{FR} = 0.85$$

CAUTION: FR does not generally equal the percentile to be computed as it does here.

The score with a rank of 17 is 9 and the score with a rank of 18 is 10. Therefore, the 85th percentile is:

$$0.85 \times (10 - 9) + 9 = 9.85$$

Let's consider the 50th percentile of the numbers 2, 3, 5, 9.

$$R = \frac{50}{100}(4 + 1) = 2.5$$

$$\text{IR} = 2$$

$$\text{FR} = 0.5$$

The score with a rank of IR is 3 and the score with a rank of IR + 1 is 5. Therefore, the 50th percentile is:

$$0.5 \times (5 - 3) + 3 = 4$$

Example 3

Finally, consider the 50th percentile of the numbers 2, 3, 5, 9, 11.

$$R = \frac{50}{100}(5 + 1) = 3$$

$$\text{IR} = 3$$

$$\text{FR} = 0$$

Whenever FR = 0, you simply find the number with rank IR. In this case, the third number is equal to 5, so the 50th percentile is 5. You will also get the right answer if you apply the general formula:

The 50th percentile equals

$$0.00 \times (9 - 5) + 5 = 5$$