Orthonormal Wavelet Basis*

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An orthonormal wavelet basis is an orthonormal basis of the form

$$B = \left\{ 2^{\frac{j}{2}} \psi \left(2^j t - k \right) \mid j \in \mathbb{Z} \ \wedge \ k \in \mathbb{Z} \right\} \tag{1}$$

The function $\psi(t)$ is called the **wavelet**.

The problem is how to find a function $\psi(t)$ so that the set B is an orthonormal set.

Example 1: Haar Wavelet

The Haar basis (described by Haar in 1910) is an orthonormal basis with wavelet $\psi(t)$

$$\psi(t) = \begin{cases} 1 \ \text{ if } \ 0 \le t \le 1/2 \\ -1 \ \text{ if } \ 1/2 \le t \le 1 \\ 0 \ \text{ otherwise} \end{cases} \tag{2}$$

For the Haar wavelet, it is easy to verify that the set B is an orthonormal set (Figure 1).

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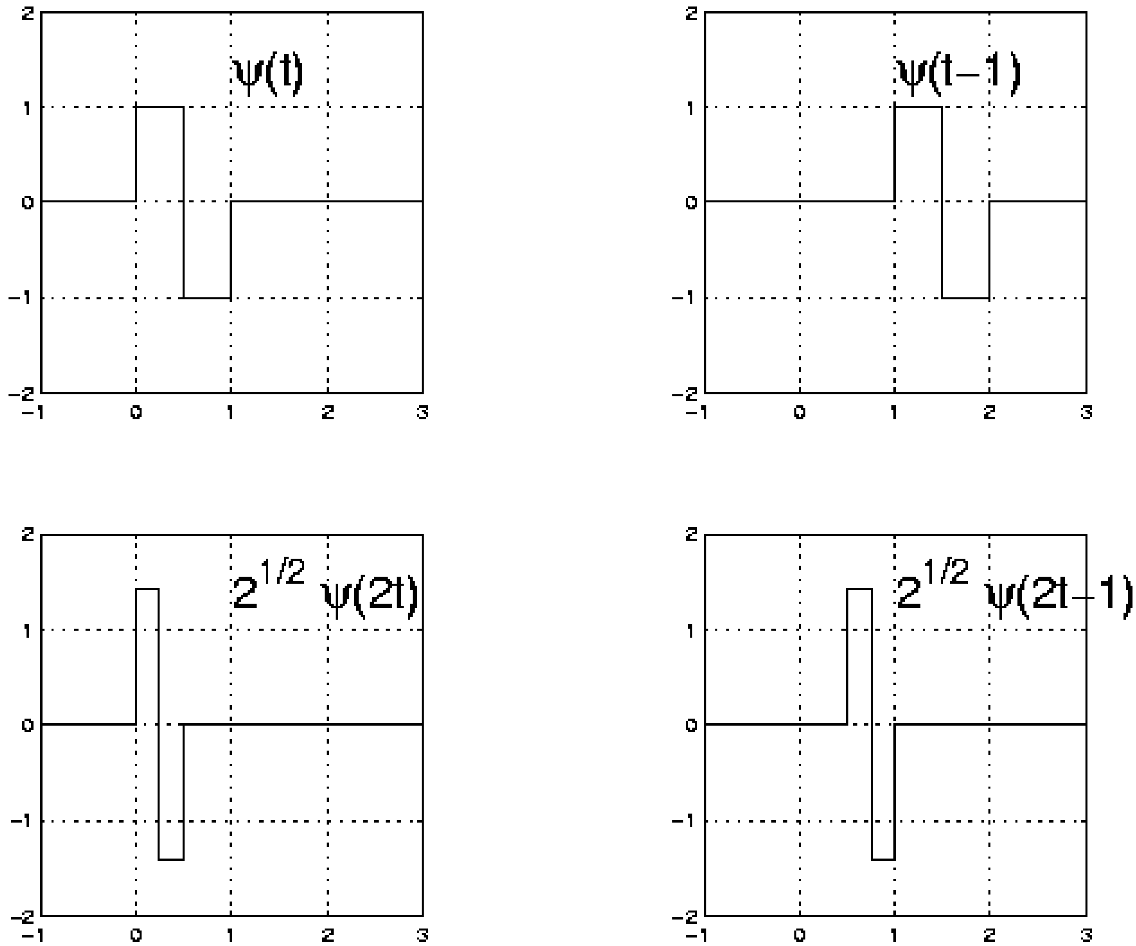

Figure 1

Notation:

$$\psi_{j,k}(t) = 2^{\frac{j}{2}} \psi\left(2^j t - k\right)$$

where j is an index of **scale** and k is an index of **location**.

If B is an orthonormal set then we have the wavelet series.

Wavelet series

$$x(t) = \sum_{j=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} d(j,k)\, \psi_{j,k}(t) \quad (3)$$

$$d(j,k) = \int_{-\infty}^{\infty} x(t)\, \psi_{j,k}(t)\, dt$$