Discrete-Time Processing of CT Signals*

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Abstract

The module will provide analysis and examples of how a continuous-time signal is converted to a digital signal and processed.

1 DT Processing of CT Signals

DSP System

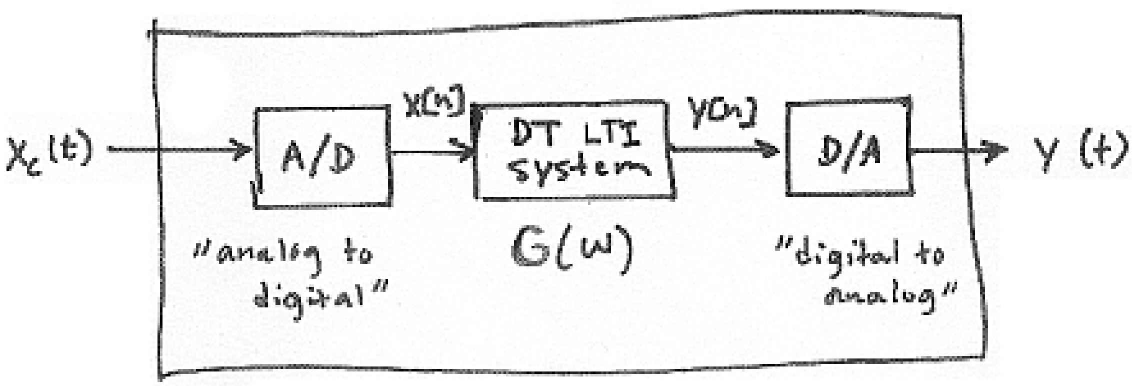

Figure 1

1.1 Analysis

$$Y_c(\Omega) = H_{\text{LP}}(\Omega)\, Y(\Omega T) \tag{1}$$

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where we know that $Y(\omega) = X(\omega)\,G(\omega)$ and $G(\omega)$ is the frequency response of the DT LTI system. Also, remember that

$$\omega \equiv \Omega T$$

So,

$$Y_c(\Omega) = H_{\text{LP}}(\Omega)\,G(\Omega T)\,X(\Omega T) \tag{2}$$

where $Y_c(\Omega)$ and $H_{\text{LP}}(\Omega)$ are CTFTs and $G(\Omega T)$ and $X(\Omega T)$ are DTFTs.

NOTE:

$$X(\omega) = \frac{2\pi}{T}\sum_{k=-\infty}^{\infty} X_c\left(\frac{\omega - 2\pi k}{T}\right)$$

OR

$$X(\Omega T) = \frac{2\pi}{T}\sum_{k=-\infty}^{\infty} X_c(\Omega - k\Omega_s)$$

Therefore our final output signal, $Y_c(\Omega)$, will be:

$$Y_c(\Omega) = H_{\text{LP}}(\Omega)\,G(\Omega T)\left(\frac{2\pi}{T}\sum_{k=-\infty}^{\infty} X_c(\Omega - k\Omega_s)\right) \tag{3}$$

Now, if $X_c(\Omega)$ is bandlimited to $\left[-\frac{\Omega_s}{2}, \frac{\Omega_s}{2}\right]$ and we use the usual lowpass reconstruction filter in the D/A, Figure 2:

Figure 2

Then,

$$Y_c(\Omega) = \begin{cases} G(\Omega T)\,X_c(\Omega) & \text{if } |\Omega| < \frac{\Omega_s}{2} \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

1.2 Summary

For bandlimited signals sampled at or above the Nyquist rate, we can relate the input and output of the DSP system by:

$$Y_c(\Omega) = G_{\text{eff}}(\Omega)\,X_c(\Omega) \tag{5}$$

where

$$G_{\text{eff}}(\Omega) = \begin{cases} G(\Omega T) & \text{if } |\Omega| < \frac{\Omega_s}{2} \\ 0 & \text{otherwise} \end{cases}$$

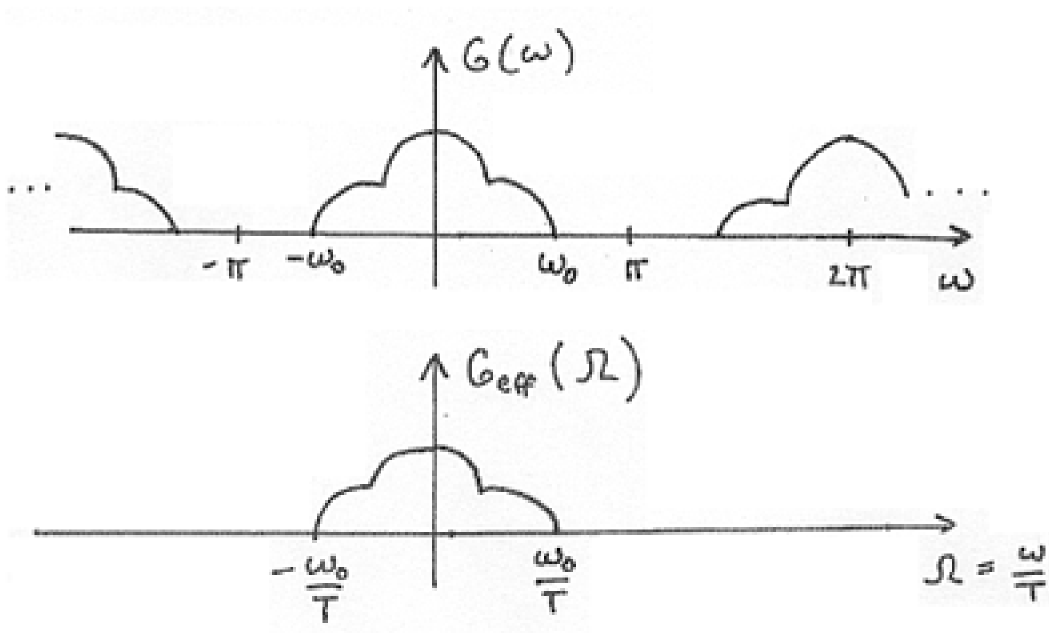

Figure 3

1.2.1 Note

$G_{\text{eff}}(\Omega)$ is LTI if and only if the following two conditions are satisfied:

1. $G(\omega)$ is LTI (in DT).
2. $X_c(T)$ is bandlimited and sampling rate equal to or greater than Nyquist. For example, if we had a simple pulse described by

$$X_c(t) = u(t - T_0) - u(t - T_1)$$

where $T_1 > T_0$. If the sampling period $T > T_1 - T_0$, then some samples might "miss" the pulse while others might not be "missed." This is what we term **time-varying behavior**.

Example 1

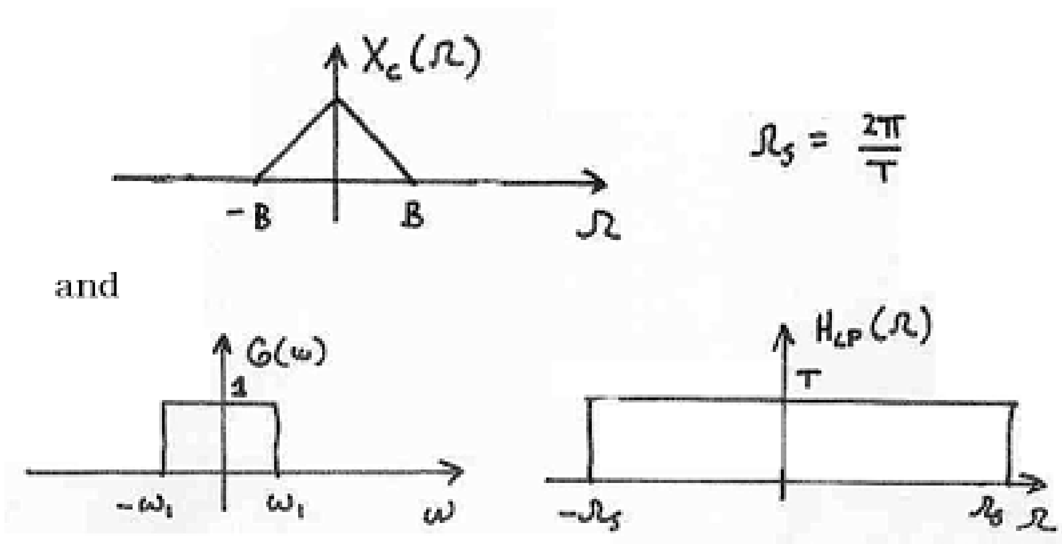

Figure 4

If $\frac{2\pi}{T} > 2B$ and $\omega_1 < BT$, determine and sketch $Y_c(\Omega)$ using Figure 4.

2 Application: 60Hz Noise Removal

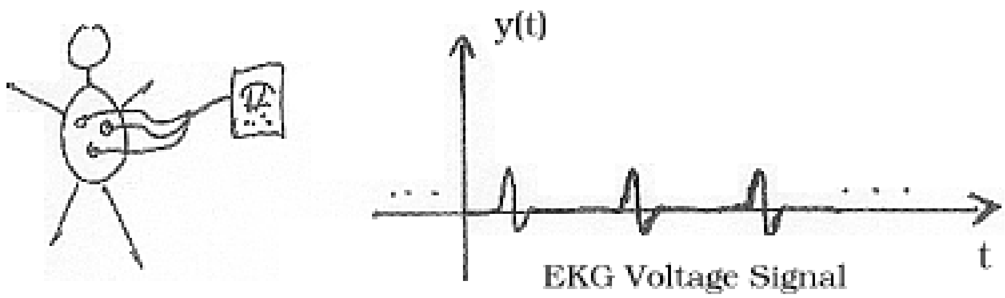

Figure 5

Unfortunately, in real-world situations electrodes also pick up ambient 60 Hz signals from lights, computers, *etc.*. In fact, usually this "60 Hz noise" is much greater in amplitude than the EKG signal shown in Figure 5. Figure 6 shows the EKG signal; it is barely noticeable as it has become overwhelmed by noise.

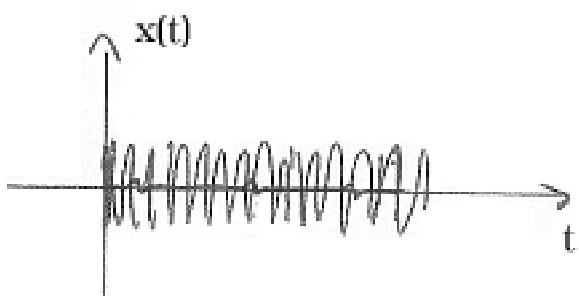

Figure 6: Our EKG signal, $y(t)$, is overwhelmed by noise.

2.1 DSP Solution

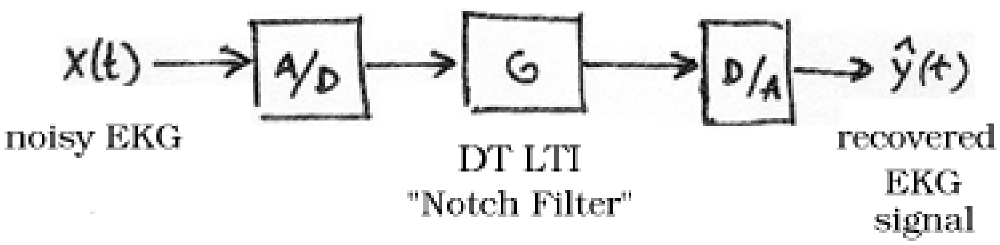

Figure 7

Figure 8

2.2 Sampling Period/Rate

First we must note that $|Y(\Omega)|$ is **bandlimited** to ±60 Hz. Therefore, the minimum rate should be 120 Hz. In order to get the best results we should set

$$f_s = 240\text{Hz}$$

.

$$\Omega_s = 2\pi \times \left(240 \frac{\text{rad}}{s}\right)$$

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Figure 9

2.3 Digital Filter

Therefore, we want to design a digital filter that will remove the 60Hz component and preserve the rest.

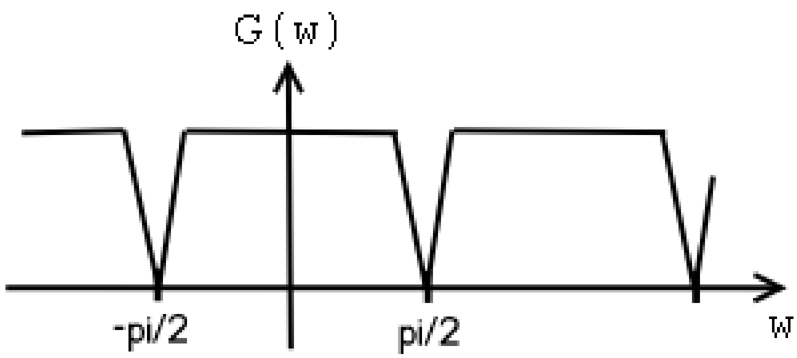

Figure 10