DISCRETE-TIME PROCESSING OF CT SIGNALS *

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Abstract

The module will provide analysis and examples of how a continuous-time signal is converted to a digital signal and processed.

1 DT Processing of CT Signals



Figure 1

1.1 Analysis

$$Y_c(\Omega) = H_{\rm LP}(\Omega) Y(\Omega T) \tag{1}$$

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where we know that $Y(\omega) = X(\omega) G(\omega)$ and $G(\omega)$ is the frequency response of the DT LTI system. Also, remember that

$$\omega \equiv \Omega T$$

So,

$$Y_{c}(\Omega) = H_{\rm LP}(\Omega) G(\Omega T) X(\Omega T)$$
⁽²⁾

where $Y_{c}(\Omega)$ and $H_{LP}(\Omega)$ are CTFTs and $G(\Omega T)$ and $X(\Omega T)$ are DTFTs.

NOTE:

$$X(\omega) = \frac{2\pi}{T} \sum_{k=-\infty}^{\infty} X_c \left(\frac{\omega - 2\pi k}{T}\right)$$

OR

$$X(\Omega T) = \frac{2\pi}{T} \sum_{k=-\infty}^{\infty} X_c \left(\Omega - k\Omega_s\right)$$

Therefore our final output signal, $Y_{c}(\Omega)$, will be:

$$Y_{c}(\Omega) = H_{\rm LP}(\Omega) G(\Omega T) \left(\frac{2\pi}{T} \sum_{k=-\infty}^{\infty} X_{c}(\Omega - k\Omega_{s})\right)$$
(3)

Now, if $X_c(\Omega)$ is bandlimited to $\left[-\frac{\Omega_s}{2}, \frac{\Omega_s}{2}\right]$ and we use the usual lowpass reconstruction filter in the D/A, Figure 2:

Image not finished

Figure 2

Then,

$$Y_{c}(\Omega) = \begin{cases} G(\Omega T) X_{c}(\Omega) & \text{if } |\Omega| < \frac{\Omega_{s}}{2} \\ 0 & \text{otherwise} \end{cases}$$
(4)

1.2 Summary

For bandlimited signals sampled at or above the Nyquist rate, we can relate the input and output of the DSP system by:

$$Y_c(\Omega) = G_{\text{eff}}(\Omega) X_c(\Omega) \tag{5}$$

where

$$G_{\text{eff}}\left(\Omega\right) = \begin{cases} G\left(\Omega T\right) & \text{if } |\Omega| < \frac{\Omega_{s}}{2} \\ 0 & \text{otherwise} \end{cases}$$



1.2.1 Note

 $G_{\text{eff}}(\Omega)$ is LTI if and only if the following two conditions are satisfied:

- 1. $G(\omega)$ is LTI (in DT).
- 2. $X_c(T)$ is bandlimited and sampling rate equal to or greater than Nyquist. For example, if we had a simple pulse described by

$$X_{c}(t) = u(t - T_{0}) - u(t - T_{1})$$

where $T_1 > T_0$. If the sampling period $T > T_1 - T_0$, then some samples might "miss" the pulse while others might not be "missed." This is what we term **time-varying behavior**.

Example 1





If $\frac{2\pi}{T} > 2B$ and $\omega_1 < BT$, determine and sketch $Y_c(\Omega)$ using Figure 4.

2 Application: 60Hz Noise Removal



Unfortunately, in real-world situations electrodes also pick up ambient 60 Hz signals from lights, computers, *etc.*. In fact, usually this "60 Hz noise" is much greater in amplitude than the EKG signal shown in Figure 5. Figure 6 shows the EKG signal; it is barely noticeable as it has become overwhelmed by noise.



Figure 6: Our EKG signal, y(t), is overwhelmed by noise.

2.1 DSP Solution



2.2 Sampling Period/Rate

First we must note that $|Y(\Omega)|$ is **bandlimited** to ± 60 Hz. Therefore, the minimum rate should be 120 Hz. In order to get the best results we should set

$$f_s = 240 \text{Hz}$$

.

$$\Omega_s = 2\pi \times \left(240 \frac{\mathrm{rad}}{s}\right)$$

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Figure 9

2.3 Digital Filter

Therefore, we want to design a digital filter that will remove the 60Hz component and preserve the rest.



