Sampling CT Signals: A Frequency Domain Perspective*

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Abstract

The module will provide an introduction to sampling a signal in the frequency domain and go through a basic example.

1 Understanding Sampling in the Frequency Domain

We want to relate $x_c(t)$ directly to $x[n]$. Compute the CTFT of

$$x_s(t) = \sum_{n=-\infty}^{\infty} x_c(nT)\,\delta(t-nT)$$

$$\begin{aligned} X_s(\Omega) &= \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} x_c(nT)\,\delta(t-nT)\,e^{(-i)\Omega t}dt \\ &= \sum_{n=-\infty}^{\infty} x_c(nT) \int_{-\infty}^{\infty} \delta(t-nT)\,e^{(-i)\Omega t}dt \\ &= \sum_{n=-\infty}^{\infty} x[n]\,e^{(-i)\Omega nT} \\ &= \sum_{n=-\infty}^{\infty} x[n]\,e^{(-i)\omega n} \\ &= X(\omega) \end{aligned} \quad (1)$$

where $\omega \equiv \Omega T$ and $X(\omega)$ is the DTFT of $x[n]$.

NOTE:

$$X_s(\Omega) = \frac{1}{T}\sum_{k=-\infty}^{\infty} X_c(\Omega - k\Omega_s)$$

$$\begin{aligned} X(\omega) &= \frac{1}{T}\sum_{k=-\infty}^{\infty} X_c(\Omega - k\Omega_s) \\ &= \frac{1}{T}\sum_{k=-\infty}^{\infty} X_c\left(\frac{\omega-2\pi k}{T}\right) \end{aligned} \quad (2)$$

where this last part is 2π-periodic.

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1.1 Sampling

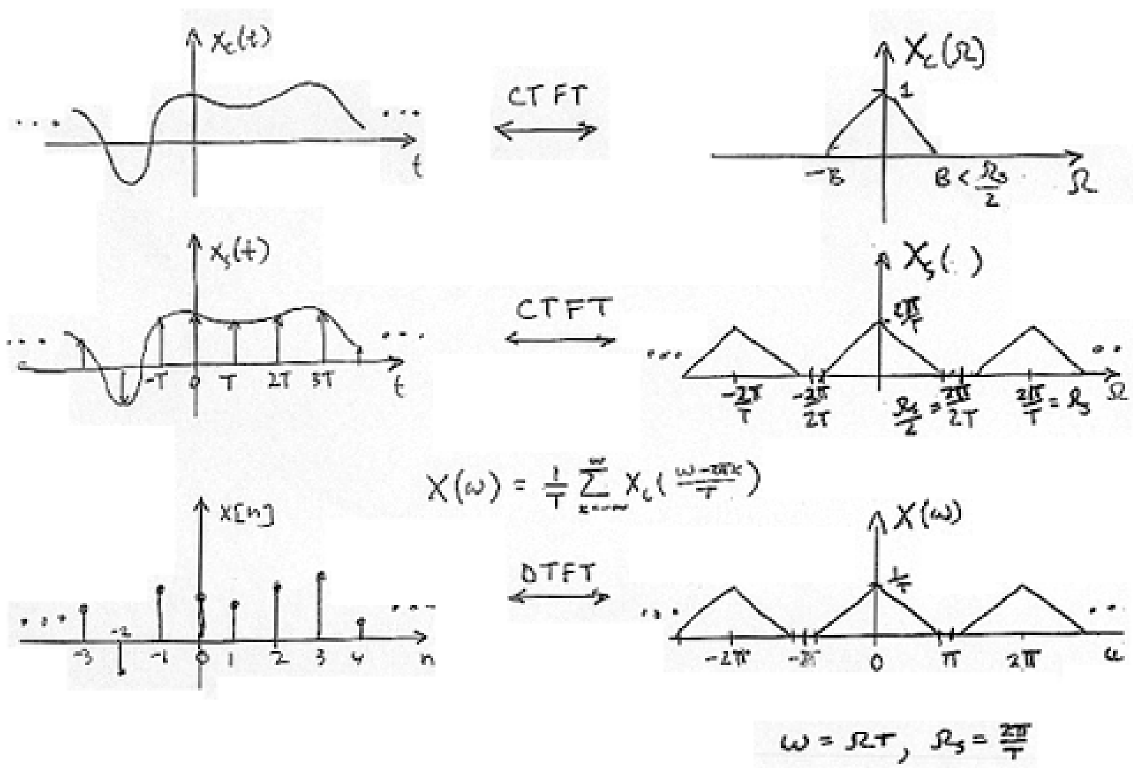

Figure 1

Example 1: Speech
Speech is intelligible if bandlimited by a CT lowpass filter to the band ±4 kHz. We can sample speech as slowly as _____?

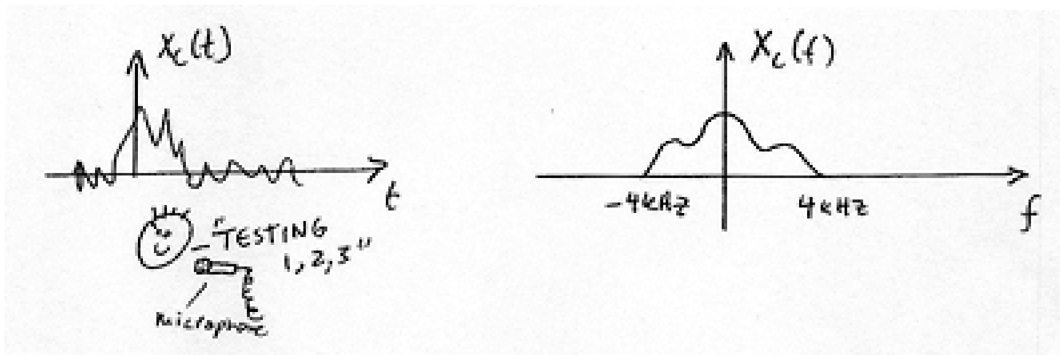

Figure 2

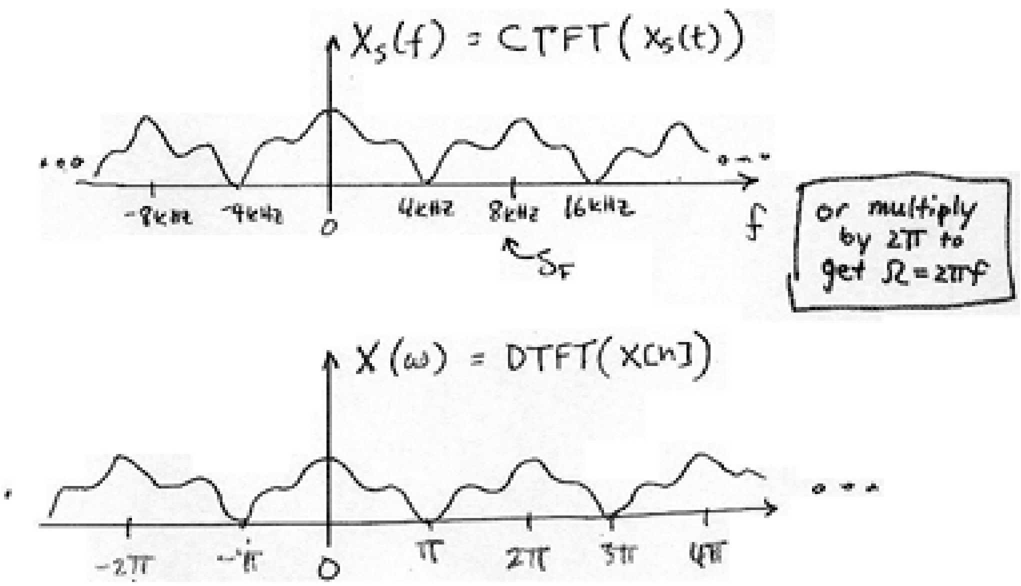

Figure 3: Note that there is no mention of T or Ω_s!

2 Relating x[n] to sampled x(t)

Recall the following equality:

$$x_s(t) = \sum_{nn} x(nT)\,\delta(t - nT)$$

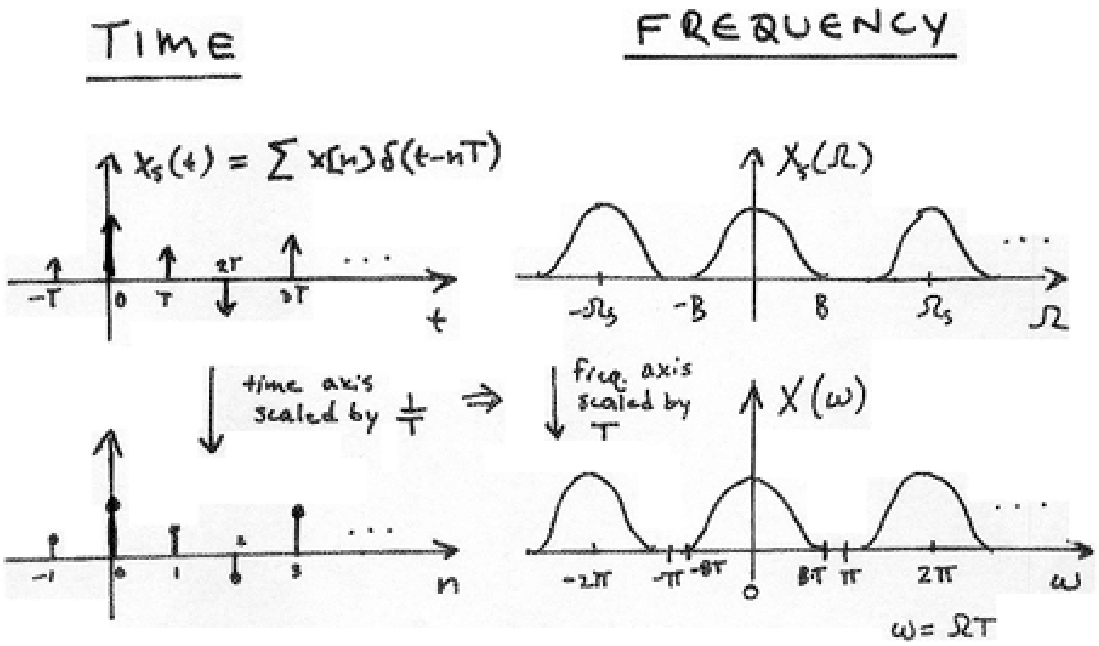

Figure 4

Recall the CTFT relation:

$$x(\alpha t) \leftrightarrow \frac{1}{\alpha} X\left(\frac{\Omega}{\alpha}\right) \tag{3}$$

where α is a scaling of time and $\frac{1}{\alpha}$ is a scaling in frequency.

$$X_s(\Omega) \equiv X(\Omega T) \tag{4}$$