IIR Filter Structures*

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Abstract

The Direct Forms I and II, the Transpose Form, Parallel Form, and Cascade Form are the most common basic structures for implementing IIR filters.

IIR (Infinite Impulse Response) filter structures must be recursive (use feedback); an infinite number of coefficients could not otherwise be realized with a finite number of computations per sample.

$$H(z) = \frac{N(z)}{D(z)} = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + \dots + b_M z^{-M}}{1 + a_1 z^{-1} + a_2 z^{-2} + \dots + a_N z^{-N}}$$

The corresponding time-domain difference equation is

$$y(n) = (-(a_1 y(n-1))) - a_2 y(n-2) + \dots - a_N y(n-N) + b_0 x(0) + b_1 x(n-1) + \dots + b_M x(n-M)$$

1 Direct-form I IIR Filter Structure

The difference equation above is implemented directly as written by the Direct-Form I IIR Filter Structure.

*Version 1.2: Dec 27, 2004 7:58 am -0600

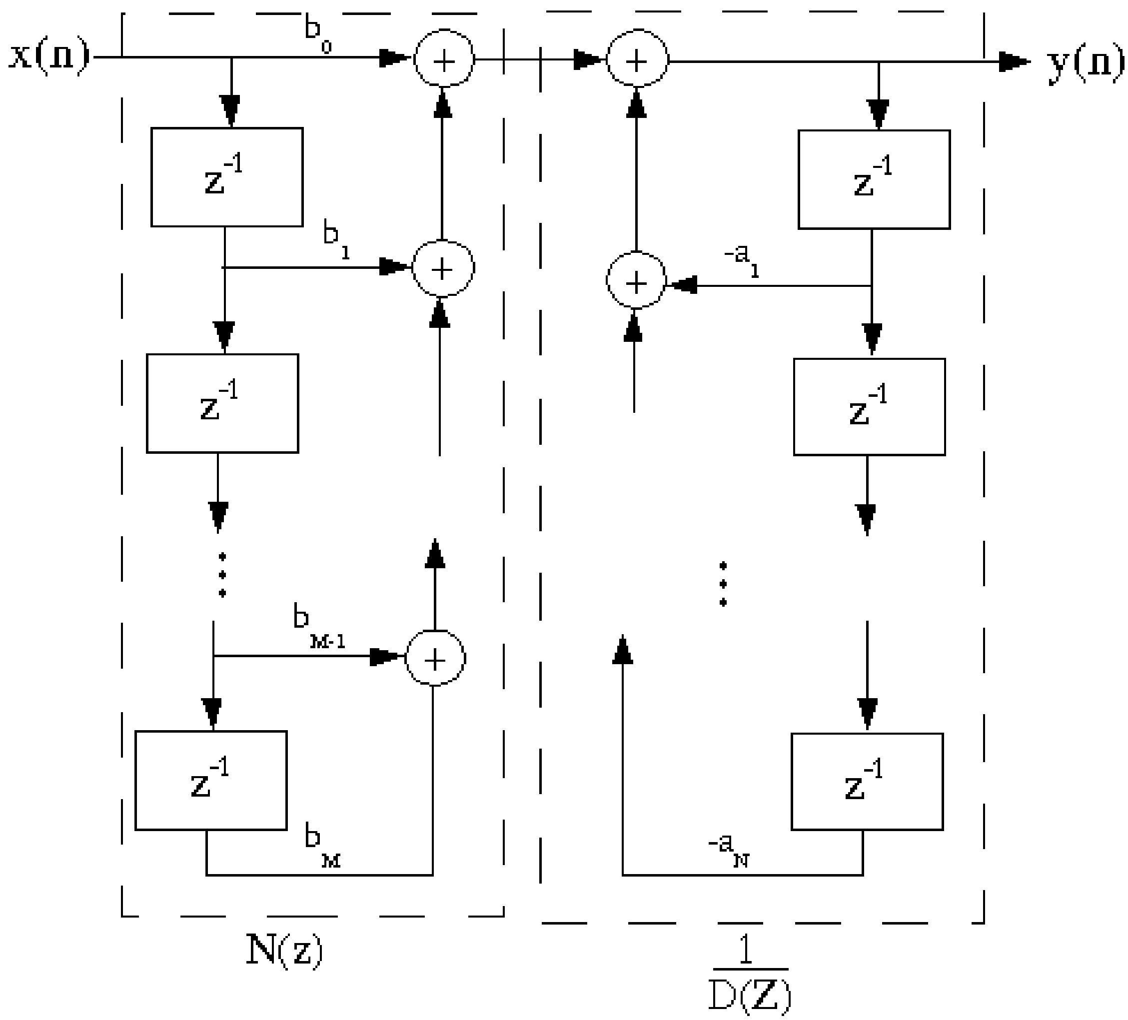

Figure 1

Note that this is a cascade of two systems, $N(z)$ and $\frac{1}{D(z)}$. If we reverse the order of the filters, the overall system is unchanged: The memory elements appear in the middle and store identical values, so they can be combined, to form the Direct-Form II IIR Filter Structure.

2 Direct-Form II IIR Filter Structure

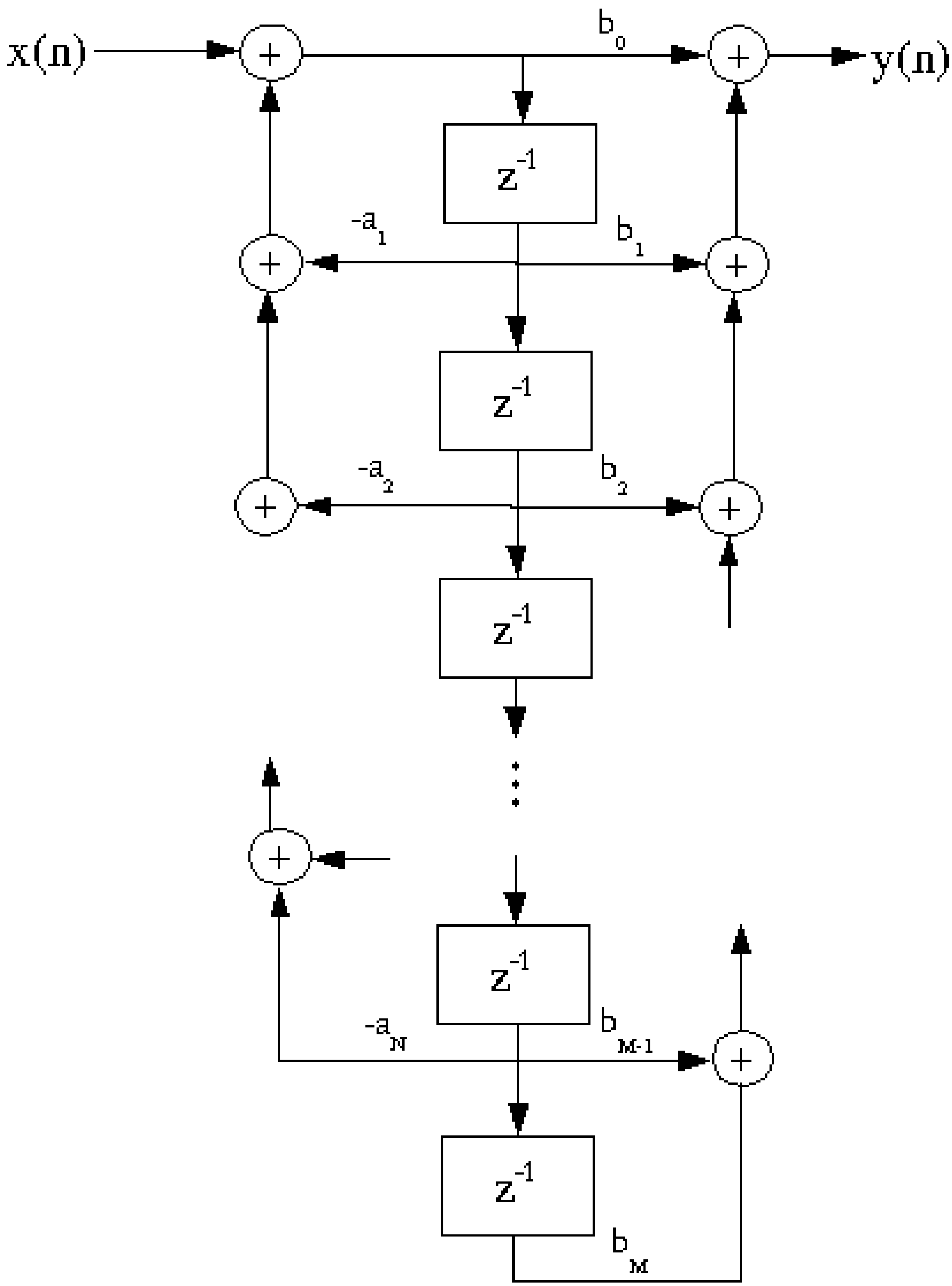

Figure 2

This structure is **canonic**: (i.e., it requires the minimum number of memory elements).

Flowgraph reversal gives the

3 Transpose-Form IIR Filter Structure

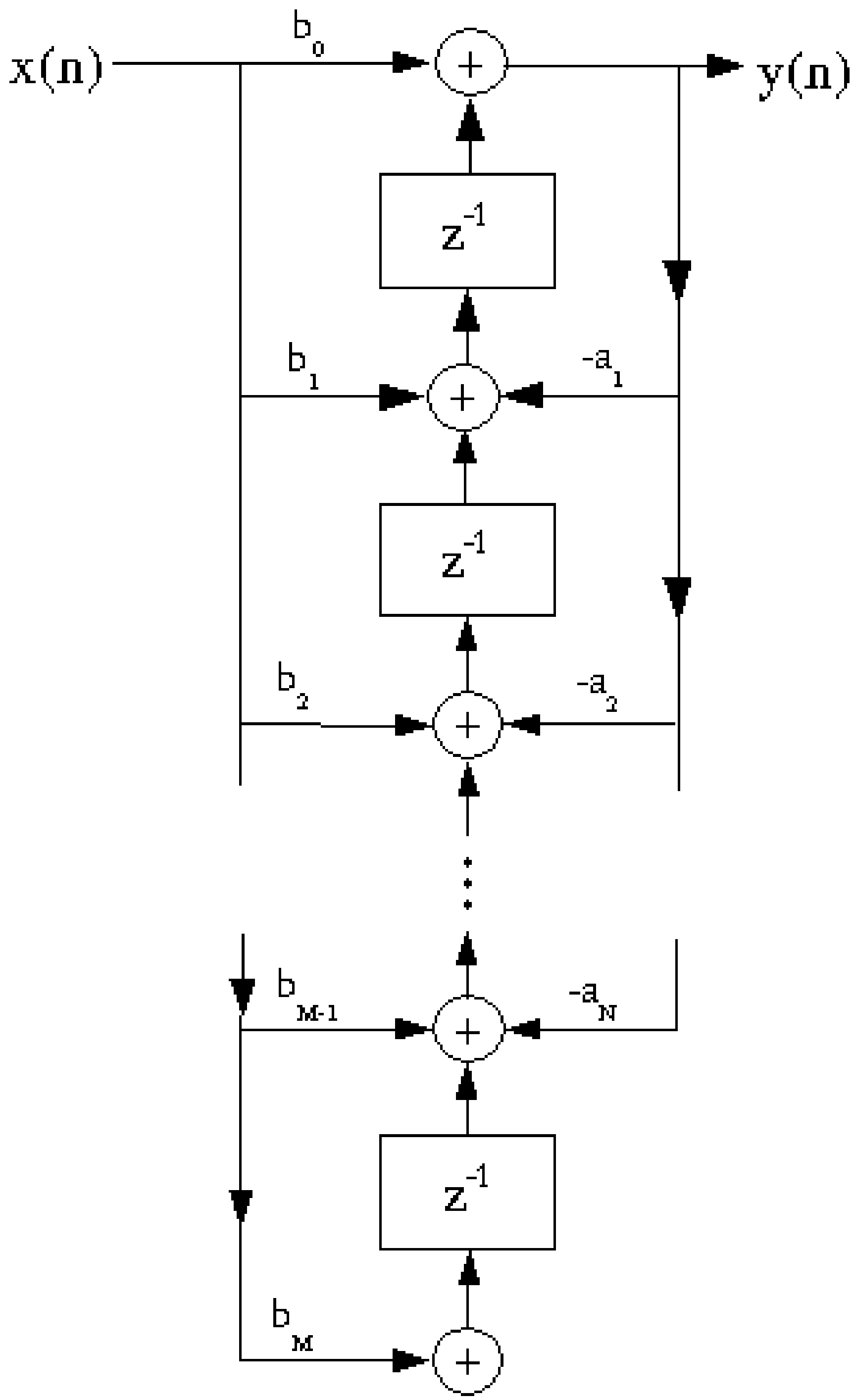

Figure 3

Usually we design IIR filters with $N = M$, but not always.

Obviously, since all these structures have identical frequency response, filter structures are not unique. We consider many different structures because

1. Depending on the technology or application, one might be more convenient than another
2. The response in a practical realization, in which the data and coefficients must be **quantized**, may differ substantially, and some structures behave much better than others with quantization.

The Cascade-Form IIR filter structure is one of the least sensitive to quantization, which is why it is the most commonly used IIR filter structure.

4 IIR Cascade Form

The numerator and denominator polynomials can be factored

$$H(z) = \frac{b_0 + b_1 z^{-1} + \dots + b_M z^{-m}}{1 + a_1 z^{-1} + \dots + a_N z^{-N}} = \frac{b_0 \prod_{k=1}^{M} z - z_k}{z^{M-N} \prod_{i=1}^{N} z - p_k}$$

and implemented as a cascade of short IIR filters.

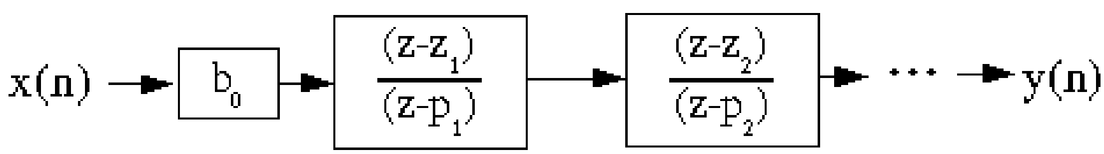

Figure 4

Since the filter coefficients are usually real yet the roots are mostly complex, we actually implement these as second-order sections, where comple-conjugate pole and zero pairs are combined into second-order sections with real coefficients. The second-order sections are usually implemented with either the Direct-Form II or Transpose-Form structure.

5 Parallel form

A rational transfer function can also be written as

$$\frac{b_0 + b_1 z^{-1} + \dots + b_M z^{-m}}{1 + a_1 z^{-1} + \dots + a_N z^{-N}} = {c_0}' + {c_1}' z^{-1} + \dots + \frac{A_1}{z - p_1} + \frac{A_2}{z - p_2} + \dots + \frac{A_N}{z - p_N}$$

which by linearity can be implemented as

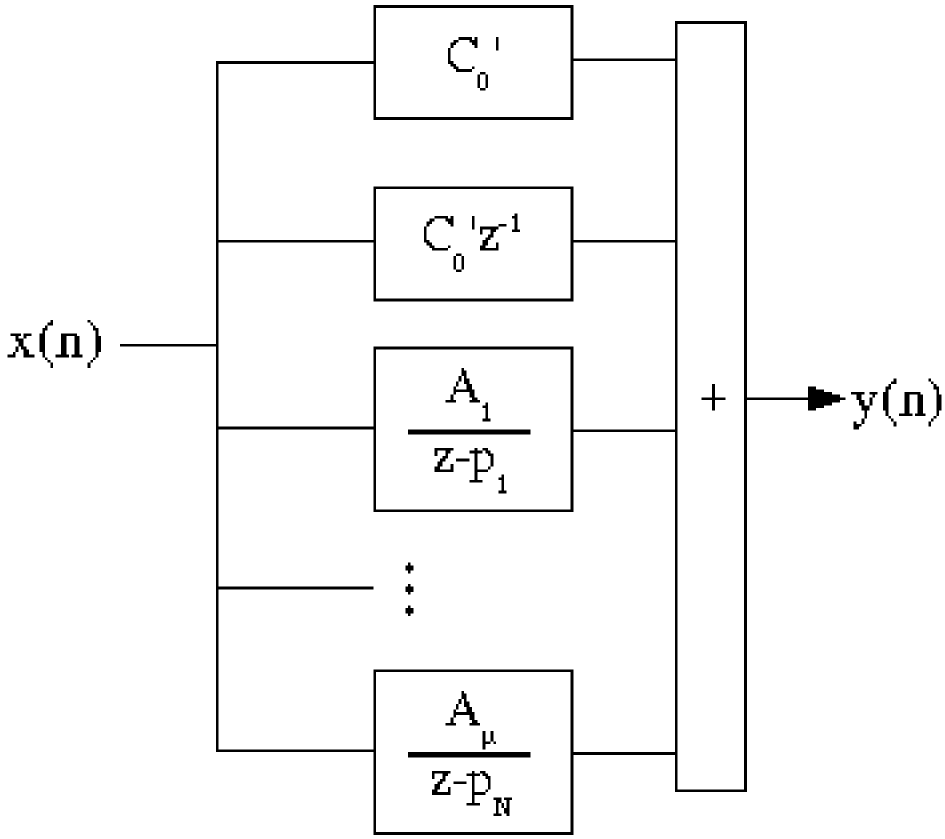

Figure 5

As before, we combine complex-conjugate pole pairs into second-order sections with real coefficients.

The cascade and parallel forms are of interest because they are much less sensitive to coefficient quantization than higher-order structures, as analyzed in later modules in this course.

6 Other forms

There are many other structures for IIR filters, such as wave digital filter structures, lattice-ladder, all-pass-based forms, and so forth. These are the result of extensive research to find structures which are computationally efficient **and** insensitive to quantization error. They all represent various tradeoffs; the best choice in a given context is not yet fully understood, and may never be.