Analytic Solutions are Important in Introductory Physics Courses*

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Abstract

A brief summary of why undergraduate science and engineering students should focus on analytic solutions rather than numerical solutions.

1 The Importance of Finding Analytic Solutions to Problems

Overview (Section 1.1: Overview) Scenario (Section 1.2: Scenario) Impossible? (Section 1.3: What if an Analytic Solution is Impossible?) Top (Section 1: The Importance of Finding Analytic Solutions to Problems)

1.1 Overview

The importance of finding analytic solutions to assigned problems is (or should be) emphasized in physics courses. Many students perform poorly on assigned problems simply because they generate numerical solutions without first achieving an analytic solution. So, it seems to be worthwhile to emphasize the importance in a different way and by a different medium than classroom discussion. Therefore this module.

In my own courses, analytic solutions are encouraged by the grading scheme. Full credit is given for a correctly derived analytic solution to any assigned problem, whether or not a numerical solution is presented. Only if the problem statement **requires** a numerical value of the solution is the numerical value part of the credit awarded. A purely numerical solution, even if correct, never receives full credit, and may receive zero credit if there is no hint of analysis in the student's work.

What is meant by an "analytic solution"? There are several short answers to this question. One is: "Always completely solve the problem **before** substituting any numerical values." Another is: "Solve the problem 'by the symbols' instead of 'by the numbers'." These short answers deserve some added details.

Every assigned problem is to be addressed by applying to the problem one or more of the fundamental relationships of the physics we have studied. Each problem provides one or more items of known quantities, the data of the problem. The data may or may not be given numerical values. If data are presented only as numerical values, it is the responsibility of the student to assign symbols that algebraically represent each item of data. Every problem asks that one or more initially unknown quantities be determined. If the problem does not provide algebraic symbols for unknown quantities, it is again the student's responsibility to assign symbols to them.

An analytic solution for an unknown quantity is an equation that explicitly states how the unknown quantity, isolated by itself on one side of an equal sign, depends upon the symbols assigned to data. The

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solution should be written in the simplest form available. The solution will often also contain mathematical constants (such as "π" or Cartesian unit vectors) and physical constants (such as "**g**", the local gravitational field strength). An analytic solution does not contain other unknown quantities. Sometimes it is useful in simplifying a solution to define a symbol for a collection of known quantities. It is OK for such a symbol to appear in a solution, but only if it is clearly defined.

Overview (Section 1.1: Overview) Scenario (Section 1.2: Scenario) Impossible? (Section 1.3: What if an Analytic Solution is Impossible?) Top (Section 1: The Importance of Finding Analytic Solutions to Problems)

1.2 Scenario

Forget for a moment that I am a professor in a college course. Assume instead that I am a Project Manager for Apex Engineering, Inc., and that you are a freshly-minted engineering graduate assigned to work for me on a project. This scenario may be of some interest, because if you are reading this module you are probably an undergraduate student with a declared major in the sciences, mathematics, or engineering. So you have expressed a desire to be educated for technical work.

As your supervisor, I assign you project tasks (instead of class problems), and I am responsible for recommendations to company management regarding your salary, your continuing employment, your raises, and your promotions (instead of awarding grades). (By the way, the author actually did this kind of work in private industry for 15 years.)

Each task I assign to you contains several technical problems (usually more difficult than the ones you are given as a student). Your solutions to those problems will greatly influence my recommendations to management regarding your employment with the company. Now, ask yourself, do I want to see analytic solutions from you, or would I prefer only numerical solutions to the problems?

The answer, as you might have already guessed, is that I **definitely** want analytic solutions. (More than once, I quickly fired "engineers" who either refused to, or were incapable of, generating analytic solutions to problems. Fellow Project Managers were often quicker about this than I was.) You may wonder why the preference is so strong. There are a number of reasons.

1.2.1 Reason 1: Is the Solution Correct?

It is usually very difficult to guarantee that a numerical solution is correct. On the other hand, an analytic solution is easily checked, and one can often be **certain** that it is either correct or incorrect. About the only way to validate a numerical solution is to have several people independently perform the calculation. This is too expensive, in both time and money, to normally be done, because the project must be completed within a budget and to meet a deadline.

A Project Manager who lets incorrect solutions "out the door", will **also** soon be "out the door"!

1.2.2 Reason 2: The Data will Change

Every Project Manager knows that the data on which calculations are done is subject to change. Maybe the original foundations designed for a building are found to violate a city zoning ordinance. Maybe someone in the marketing department discovers that consumers want a smaller widget, or the competition has added a feature that our widget must also include. Or maybe the EPA has issued a new regulation that means the smokestack emissions have to be lowered.

The likelihood of getting all the way to the end of a project without some significant changes in the original data is virtually zero. (Projects usually take weeks or months, some much longer.) So, how much time and money does it cost to adjust the solutions to changes in the values of the data?

If the original calculations produced only a numerical result, and the data have changed, then the only recourse is to do the entire calculation over again from scratch. This is generally prohibitively expensive. If the original calculations produced analytic solutions, changes in the data may not affect the solution at all, or the changes may be accommodated by the solution much more quickly and easily than doing the calculations over again from the beginning.

1.2.3 Reason 3: Numerical Solutions have Little Value

In the great preponderance of problems encountered in real-world practice, a numerical solution is of little value. What is always of importance is how the solution depends upon the data, which is precisely what an analytic solution tells you. This is especially true of design projects.

Take for example a calculation of the amount of concrete needed to pour the foundations of a new building. The shape and size of the foundations may have changed six times over the course of the design project. The very first time that the numerical solution is actually needed is after everyone involved (the engineer, the Project Manager, other company management, the client, and the building inspector) has signed off on the design to actually be used, and it is time to order the concrete from the concrete vendor. Any numerical values for the amount of concrete prior to the final design are simply useless, of no value whatsoever. The original analytical solution for the amount of concrete needed may still be correct. Even if the design changes do change the original solution, if the derivation of that first solution is documented, it is probably a quick and easy task to change the derivation for a correct final solution.

1.2.4 Reason 4: Numerical Solutions Waste Effort

This is the least important reason to prefer analytic solutions, but it is significant nonetheless. A characteristic of numerical solutions is that often quantities that algebraically "cancel out" of the final analytic solution are multiplied (perhaps several times) by individual terms, then the final sum is divided by the same quantity. A simple example can be seen in the calculation of the final velocity of a particle in free fall, using the energy equation. The mass of the particle appears in the energy equation, but vanishes from the solution for the final velocity. Such quantities may be multiplied and divided several times in a numerical solution, but simply vanish from an analytic solution.

Overview (Section 1.1: Overview) Scenario (Section 1.2: Scenario) Impossible? (Section 1.3: What if an Analytic Solution is Impossible?) Top (Section 1: The Importance of Finding Analytic Solutions to Problems)

1.3 What if an Analytic Solution is Impossible?

Problems for which no analytic solution is possible are rare in introductory courses. You may see one or two like this in an introductory course in physics. On the other hand, such problems are fairly frequent in upper-level and graduate courses, and very common in actual professional practice.

Such problems are usually solved using computerized numerical techniques.

What is most difficult about problems that require numerical techniques is validating that the solution is indeed correct. This always requires **much more analysis** than is the case for problems that admit of analytic solutions. This is why you are not exposed to such problems until your analytical skills have reached a certain level.

The validation of the solution of a problem that cannot be solved analytically usually means a validation of the computer code that generates the numerical solution.

Overview (Section 1.1: Overview) Scenario (Section 1.2: Scenario) Impossible? (Section 1.3: What if an Analytic Solution is Impossible?) Top (Section 1: The Importance of Finding Analytic Solutions to Problems)