Window Design Method*

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The truncate-and-delay design procedure is the simplest and most obvious FIR design procedure.

Exercise 1 *(Solution on p. 3.)*

Is it any Good?

1 L2 optimization criterion

find $\forall n, 0 \leq n \leq M-1 : (h[n])$, maximizing the energy difference between the desired response and the actual response: i.e., find

$$min_{\text{hn}}\left\{\text{hn}, \int_{-\pi}^{\pi} (|H_d(\omega) - H(\omega)|)^2 d\omega\right\}$$

by Parseval's relationship[1]

$$min_{\text{hn}}\left\{\text{hn}, \int_{-\pi}^{\pi} (|H_d(\omega) - H(\omega)|)^2 d\omega\right\} = 2\pi \sum_{n=-\infty}^{\infty} (|h_d[n] - h[n]|)^2 = 2\pi \left(\sum_{n=-\infty}^{-1} (|h_d[n] - h[n]|)^2 + \sum_{n=0}^{M-1} (|h_d[n] - h[n]|)^2 + \sum_{n=M}^{\infty} (|h_d[n] - h[n]|)^2\right) \quad (1)$$

Since $\forall n, n < 0 n \geq M : (h[n])$ this becomes

$$min_{\text{hn}}\left\{\text{hn}, \int_{-\pi}^{\pi} (|H_d(\omega) - H(\omega)|)^2 d\omega\right\} = \sum_{h=-\infty}^{-1} (|h_d[n]|)^2 + \sum_{n=0}^{M-1} (|h[n] - h_d[n]|)^2 + \sum_{n=M}^{\infty} (|h_d[n]|)^2$$

NOTE: $h[n]$ has no influence on the first and last sums.

The best we can do is let

$$h[n] = \begin{cases} h_d[n] & \text{if } 0 \leq n \leq M-1 \\ 0 & \text{if else} \end{cases}$$

Thus $h[n] = h_d[n] w[n]$,

$$w[n] = \begin{cases} 1 & \text{if } 0 \leq n (M-1) \\ 0 & \text{if else} \end{cases}$$

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[1] "Parseval's Theorem" <http://cnx.org/content/m0047/latest/>

is **optimal** in a least-total-sqaured-error (L_2, or energy) sense!

Exercise 2 *(Solution on p. 3.)*

Why, then, is this design often considered undersirable?

For desired spectra with discontinuities, the least-square designs are poor in a minimax (worst-case, or L_∞) error sense.

2 Window Design Method

Apply a more gradual truncation to reduce "ringing" (Gibb's Phenomenon[2])

$$\forall \mathrm{n0} \leq \mathrm{n} \leq \mathrm{M} - 1\mathrm{hn} = \mathrm{hdnwn} : (\mathrm{n0} \leq \mathrm{n} \leq \mathrm{M} - 1\mathrm{hn} = \mathrm{hdnwn})$$

NOTE: $H(\omega) = H_d(\omega) * W(\omega)$

The window design procedure (except for the boxcar window) is ad-hoc and not optimal in any usual sense. However, it is very simple, so it is sometimes used for "quick-and-dirty" designs of if the error criterion is itself heurisitic.

[2]"Gibbs Phenomena" <http://cnx.org/content/m10092/latest/>

Solutions to Exercises in this Module

Solution to Exercise (p. 1)

Yes; in fact it's optimal! (in a certain sense)

Solution to Exercise (p. 2): Gibbs Phenomenon

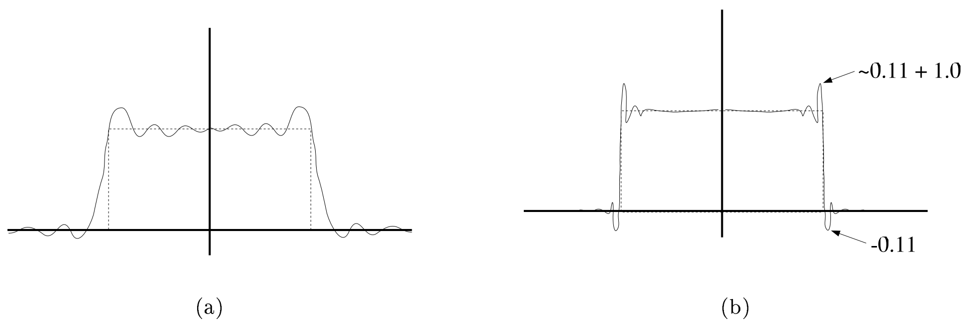

Figure 1: (a) $A(\omega)$, small M (b) $A(\omega)$, large M