Ampere's Law*

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Abstract

Ampere's Law with the displacement current is expressed in differential form.

1 Ampere's Law (with displacement current)

For a steady current flowing through a straight wire, the magnetic field at a point at a perpendicular distance r from the wire, has a value

$$B = \frac{\mu_0 I}{2\pi r}$$

If we integrate around the wire in a circle, then clearly we get

$$\begin{aligned} \oint \vec{B} \cdot d\vec{l} &= \tfrac{\mu_0 I}{2\pi r} \oint dl \\ &= \tfrac{\mu_0 I}{2\pi r} 2\pi r \\ &= \mu_0 I \end{aligned}$$

This is true for irregular paths around the wire

$$\oint \vec{B} \cdot d\vec{l} = \oint B \cdot dl \cos\theta$$

but for small $dl dl \cos\theta = r d\phi$

$$\begin{aligned} \oint \vec{B} \cdot d\vec{l} &= \oint B r d\phi \\ &= \tfrac{\mu_0 I}{2\pi} \oint d\phi \\ &= \mu_0 I \end{aligned}$$

In fact instead of current we use the surface integral of the current density J, which is the current per unit area

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \int_S \vec{J} \cdot d\vec{A}$$

Maxwell's great insight was to realize that this was incomplete. He reasoned that $\frac{d\phi_B}{dt}$ gives a $\vec{E}$ field so we should expect that $\frac{d\phi_E}{dt}$ gives a $\vec{B}$ field.

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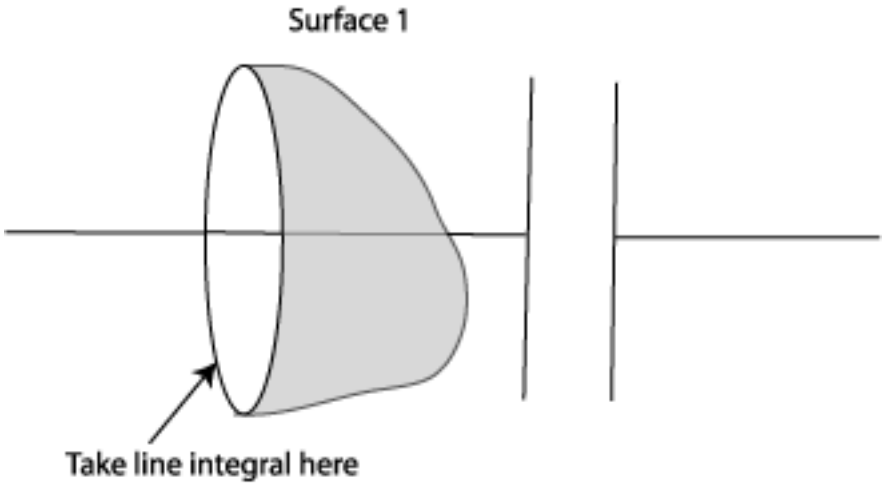

Figure 1

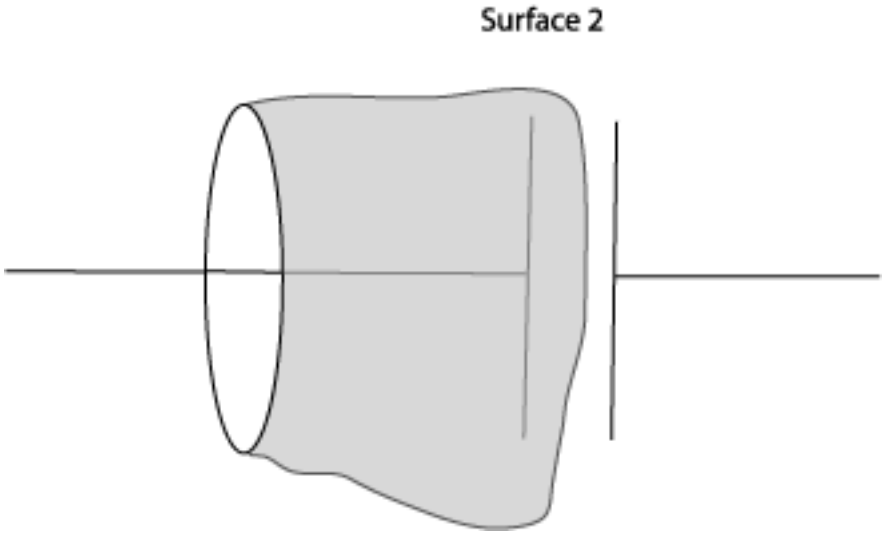

Figure 2

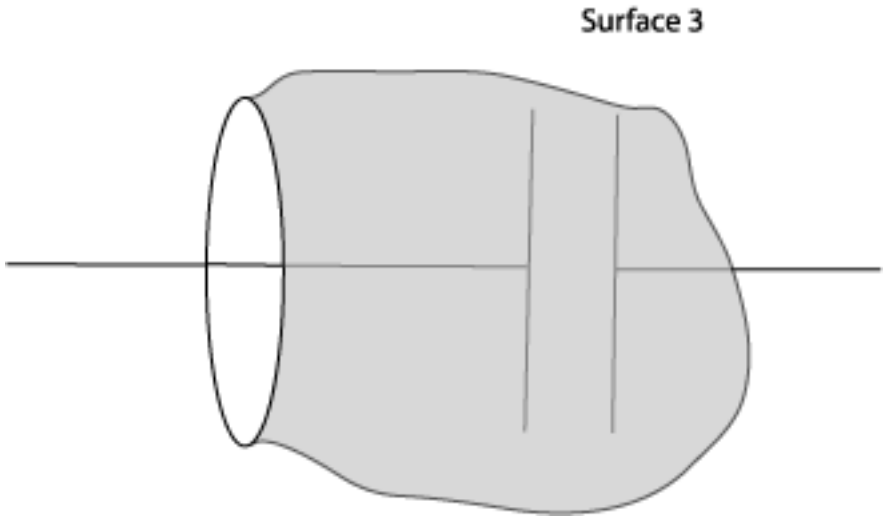

Figure 3

Think of a capacitor in a simple circuit. We can draw a surface such as shown in the figure, with "surface 1" and take the line integral around the edge of the surface. Now look at surface 2, this will have the same line integral, but now the surface integral will be different. Clearly there is something incomplete with Ampere's law as formulated above. Maxwell re wrote Ampere's law

$$\oint \vec{B} \cdot d\,\vec{l} = \mu_0 \int_S \left(\vec{J} + \epsilon_0 \frac{\partial \vec{E}}{\partial t} \right) \cdot d\,\vec{A}$$

which solves the problem.

Again it is left as an exercise to show that

$$\vec{\nabla} \times \vec{B} = \mu_0 \left(\vec{J} + \epsilon_0 \frac{\partial \vec{E}}{\partial t} \right)$$

2 Maxwell's equations

Lets recall Maxwell's equations (in free space) in differential form

$$\begin{gathered} \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \\ \vec{\nabla} \times \vec{B} = \mu_0 \left(\vec{J} + \epsilon_0 \frac{\partial \vec{E}}{\partial t} \right) \\ \vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0} \\ \vec{\nabla} \cdot \vec{B} = 0 \end{gathered}$$