CLFDS5*

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Abstract

Page 5 of 6 of a PLTL workshop on Coulomb law fields, for undergraduate physics.

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2 Problems

Work on these problems with your Peer Team members. Determine analytic solutions **before** substituting any numerical values to find a numerical solution. Each problem is solved by use of either Equation 2 ($\vec{E}\left[\vec{r}\right] = \frac{1}{4\pi\varepsilon_0}\sum_{n=1}^{N}\frac{q_n}{{r_n}^2}\hat{r}_n$) or Equation 3 ($\vec{E}\left[\vec{r}\right] = \frac{1}{4\pi\varepsilon_0}\int_{V'}\frac{\rho\hat{\eta}}{\eta^2}dV'$, or the lower-dimension analogues involving $\sigma := \frac{dq}{dS'}$ or $\lambda := \frac{dq}{dL'}$). These basic relationships should form the starting point of your solutions, although other basic relationships that you have encountered before, or perhaps have to look up, may also be needed to complete the solutions.

2.1 Problem 1

Consider a disk in the xy plane with radius $R = 3\,\text{cm}$ and constant charge density $\sigma = 3.4\frac{C}{\text{cm}^2}$, centered on the origin of coordinates. Find the electric field everywhere on the z axis.

2.2 Problem 2

Repeat Problem 1 with the charge density changed to $\sigma = \sigma_0\frac{s'}{R}$, where s' is the radial variable in the xy plane: $s' = \sqrt{{x'}^2 + {y'}^2}$, restricted to the charge distribution, and σ_0 is the charge density of Problem 1.

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[1] "Coulomb Law Fields" <http://cnx.org/content/m13116/latest/>

[2] "CLFds2" <http://cnx.org/content/m13117/latest/>

[3] "CLFds3" <http://cnx.org/content/m13118/latest/>

[4] "CLFds4" <http://cnx.org/content/m13119/latest/>

[5] "CLFds6" <http://cnx.org/content/m13121/latest/>

2.3 Problem 3

The figure below can be taken as a very crude model of a water molecule. The red disk represents the oxygen atom and the two green disks represent the hydrogen atoms in the water molecule. Given the coordinate system shown in the diagram, find the electric field everywhere on the x axis except the origin. In nature, and for an isolated water molecule, the bond distance $d \approx 1.0\,\text{Å} = 10^{-10}\text{m}$, the angle between the hydrogen atoms is $2\alpha \approx \frac{7\pi}{12} = 105\,^{\circ}$ and the charge $q \approx \frac{9}{10}e$. These values can change significantly if other molecules are in the neighborhood of the water molecule. Note the coordinate system chosen: all charges lie in the xy plane, with the x axis bisecting the bond angle 2α.

Isolated Water Molecule

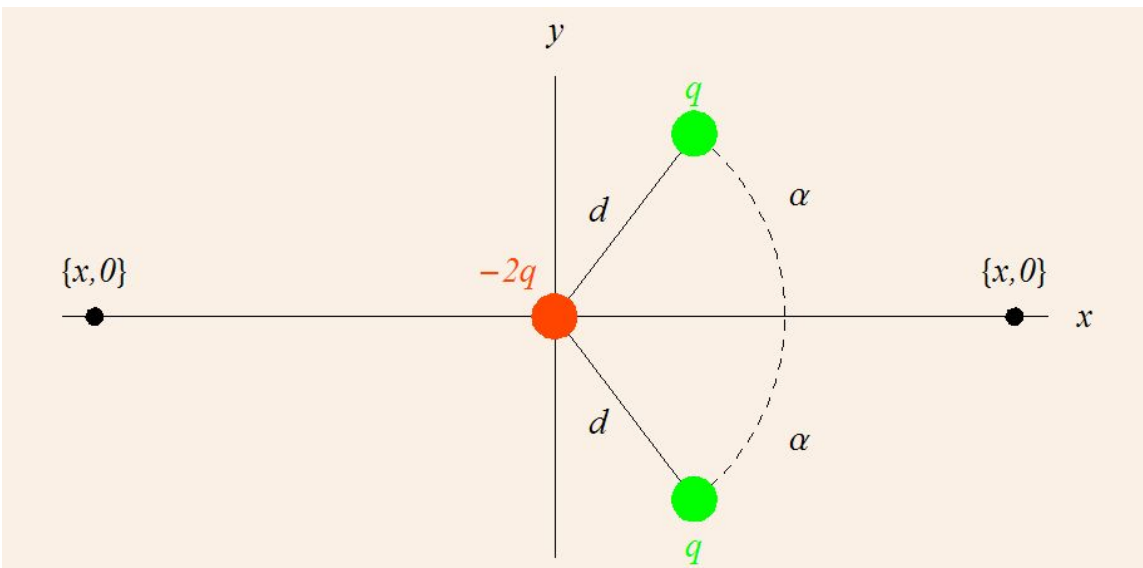

Figure 1: A crude model of the water molecule, with hydrogen atoms represented by green disks and the oxygen atom represented by a red disk. The black dots represent field points on the x axis.

2.4 Problem 4

Four equal point charges q lie at the corners of a regular tetrahedron centered on the origin, as diagrammed below. The coordinates of the point charges are $\{a\{\frac{1}{\sqrt{3}}, 0, \frac{1}{2\sqrt{6}}, \}, a\{-\frac{1}{2\sqrt{3}}, \frac{1}{2}, -\frac{1}{2\sqrt{6}}\}, a\{-\frac{1}{2\sqrt{3}}, \frac{1}{2}, -\frac{1}{2\sqrt{6}}\}, a\{0, 0, \frac{1}{2}\sqrt{\frac{3}{2}}\}\}$, where a is the edge length of the tetrahedron, which is the distance separating any two of the charges.

Tetrahedron of Identical Point Charges

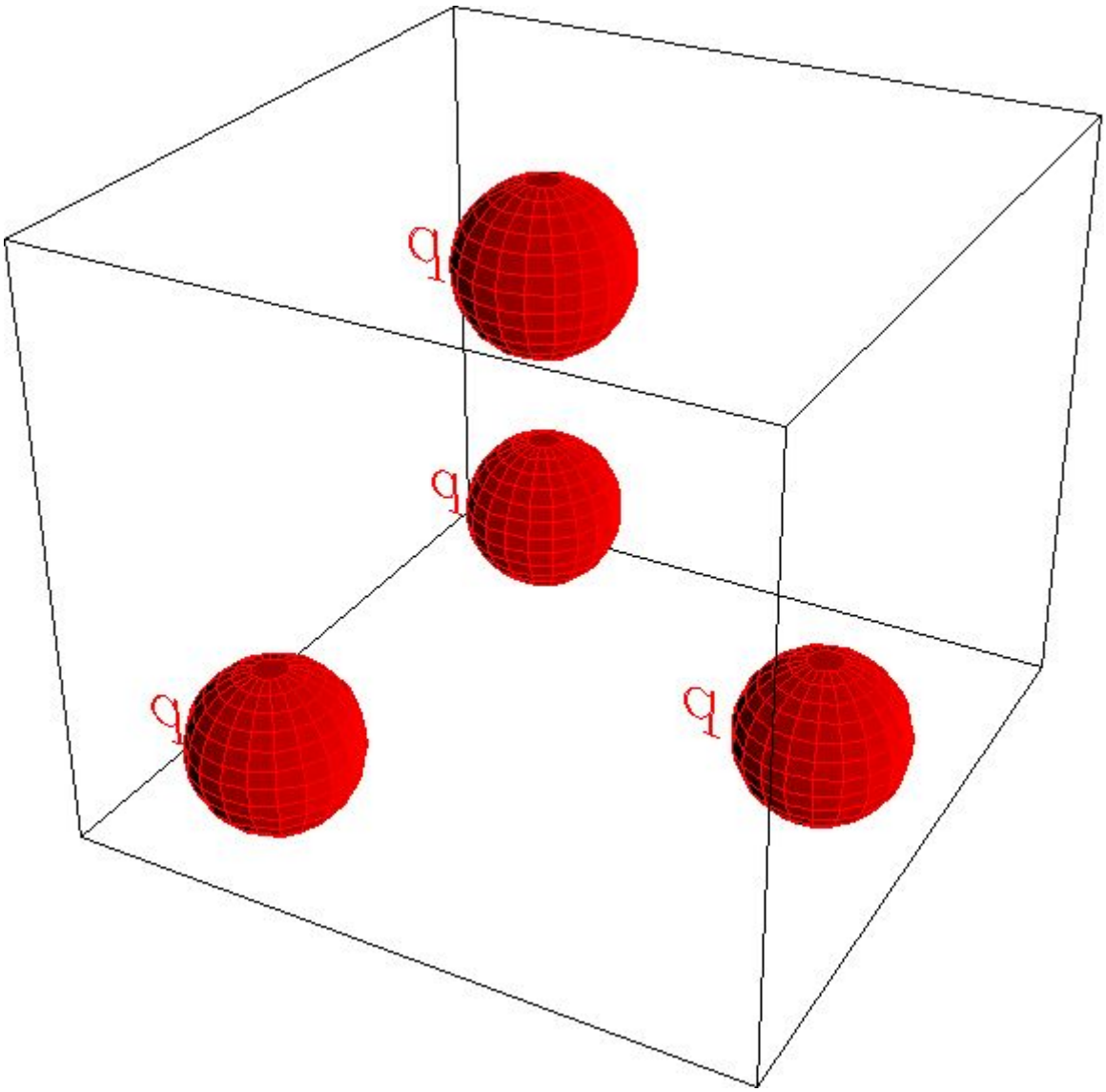

Figure 2: Four equal point charges on the vertices of a tetrahedron.

(a) Demonstrate that the electric field at the origin is zero.

Now consider the red spheres to represent hydrogen atoms, and add a nitrogen atom at the center of the tetrahedron (blue sphere). This gives us a (very, very crude) model of the ammonium ion, $NH_4{}^+$, pictured below.

Crudely Modeled Ammonium Ion

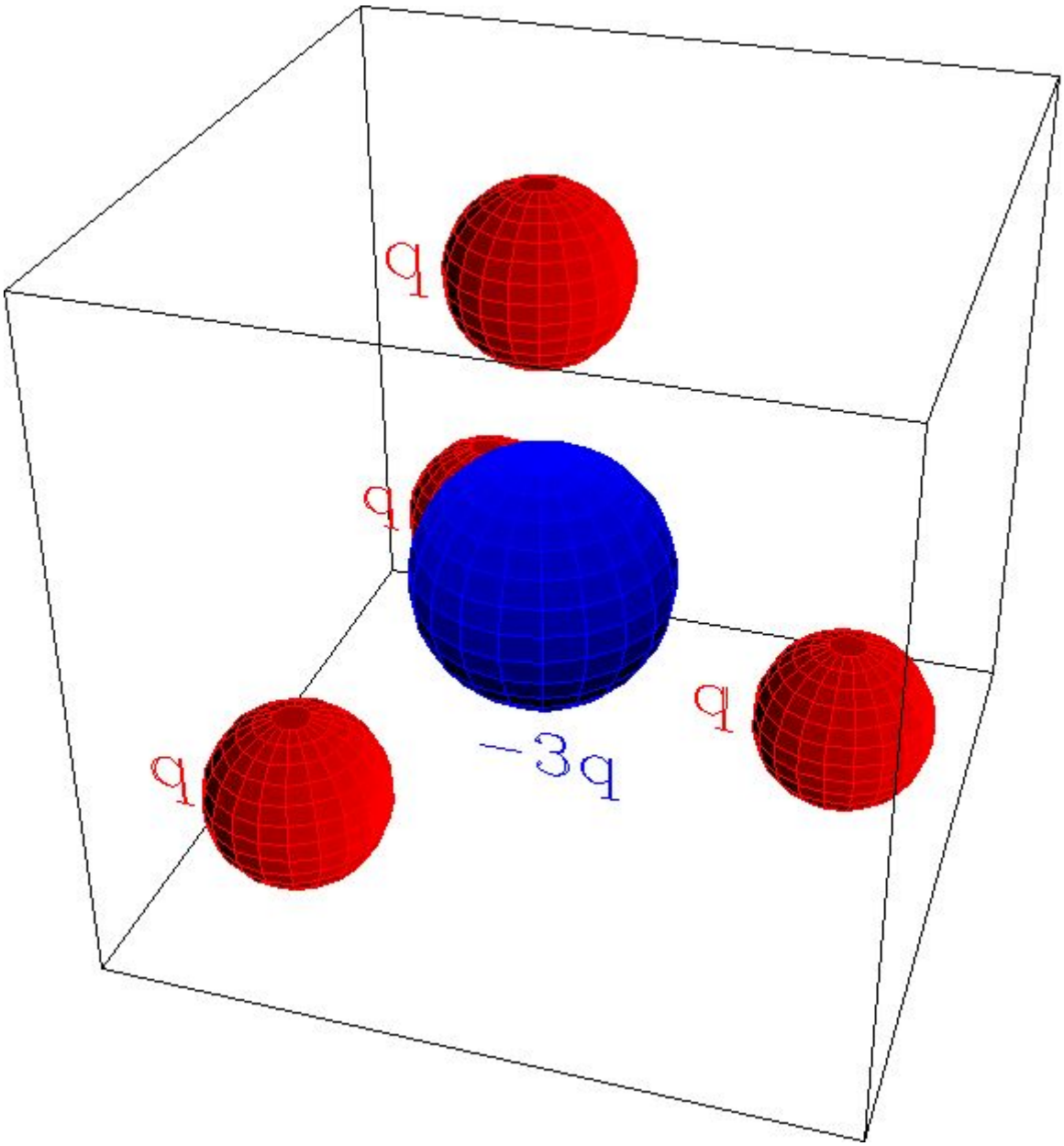

Figure 3: A very, very crude model of the ammonium ion, $NH_4{}^+$. The central blue sphere represents the nitrogen atom, the red spheres hydrogen atoms.

(b) For this model, consider a field point located at the center of any one of the four triangular sides of the tetrahedron. The sum for the electric field at this point reduces to contributions from only two of the five point source charges. Why? Which two charges sum to make a nonzero contribution? Is the direction

of this electric field pointing out of the tetrahedron or into it? What is the value of the electric field at this field point? Justify your answers.

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