Discontinuities*

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Abstract

Finding discontinuities - vertical asymptotes and holes - of rational functions

Vertical Asymptotes occur when factors in the denominator = 0 and do not cancel with factors in the numerator

- **Vertical asymptotes** are vertical lines the graph approaches
- The equation of the vertical asymptote is x = (that number which makes the denominator = 0)

Holes (**Removable Discontinuities**) occur when the factor in the denominator = 0 and it cancels with like factors in the numerator.

- Holes are open "points" so they have an x and y coordinate
- The x-value is the number that makes the cancelled factor = 0.
- The y-value is found by substituting x into the "reduced" equation (**after** cancelling) like factors.

Find the vertical asymptotes and holes (if any) for the following. Don't forget that vertical asymptotes are equations and holes are points!

Example 1
$y = \frac{1}{x}$
Vertical Asymptote: $x = 0$
Hole: None

Example 2
$y = \frac{x(x-1)}{x-1}$
Vertical Asymptote: None
Hole: (1,1) since (x-1) was cancelled, the hole is at x=1. To find the y-coordinate, plug 1 into the reduced equation: $\frac{x(x-1)}{x-1} = x = 1$

Exercise 1 ***(Solution on p. 3.)***
$y = \frac{4x+3}{x-7}$

Exercise 2 ***(Solution on p. 3.)***
$y = \frac{9x}{3-2x}$

Exercise 3 ***(Solution on p. 3.)***
$y = \frac{7}{(x-9)(x+1)}$

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Exercise 4 *(Solution on p. 3.)*

$y = \frac{7x}{2x^2 - 7x + 3}$

Exercise 5 *(Solution on p. 3.)*

$y = \frac{2x+1}{(x+5)^2}$

Exercise 6 *(Solution on p. 3.)*

$y = \frac{x+3}{x^2+25}$

Exercise 7 *(Solution on p. 3.)*

$y = \frac{x-7}{x^2+2}$

Exercise 8 *(Solution on p. 3.)*

$y = \frac{5}{|x-3|}$

Exercise 9 *(Solution on p. 3.)*

$y = \frac{4}{|x|-4}$

Exercise 10 *(Solution on p. 3.)*

$y = \frac{3\left(x^2-x-6\right)}{4(x^2-9)}$

Exercise 11 *(Solution on p. 3.)*

$y = \frac{-2\left(x^2-4\right)}{3(x^2+4x+4)}$

Exercise 12 *(Solution on p. 3.)*

$y = \frac{x^2-4}{x+2}$

Exercise 13 *(Solution on p. 3.)*

$y = \frac{x^2(x-3)}{x^2-3x}$

Exercise 14 *(Solution on p. 3.)*

$y = \frac{x^3-1}{x-1}$

Exercise 15 *(Solution on p. 4.)*

$y = \frac{2x^2-3x-5}{x^2-1}$

Solutions to Exercises in this Module

Solution to Exercise 1 (p. 1)
Vertical Asymptote: $x = 7$ since $x - 7 = 0$
Hole: None

Solution to Exercise 2 (p. 1)
Vertical Asymptote: $x = \frac{3}{2}$ since $3 - 2x = 0$, $x = \frac{3}{2}$
Hole: None

Solution to Exercise 3 (p. 1)
Vertical Asymptote: $x = 9$, $x = -1$ since $x = 9$ and $x = -1$
Hole: None

Solution to Exercise 4 (p. 2)
Vertical Asymptote: $x = \frac{1}{2}$, $x = 3$ since $2x^2 - 7x + 3 = 0$, $(2x-1)(x-3) = 0$, $2x - 1 = 0$ and $x - 3 = 0$, $x = \frac{1}{2}$ and $x = 3$
Hole: None

Solution to Exercise 5 (p. 2)
Vertical Asymptote: $x = -5$ since $(x+5)^2 = 0$, $x + 5 = 0$, $x = -5$
Hole: None

Solution to Exercise 6 (p. 2)
Vertical Asymptote: None since $x^2 + 25 = 0$, $x^2 = -25$, a number squared will never be negative
Hole: None

Solution to Exercise 7 (p. 2)
Vertical Asymptote: None since $x^2 + 2 = 0$, $x^2 = -2$ and any number squared will never be a negative number
Hole: None

Solution to Exercise 8 (p. 2)
Vertical Asymptote: $x = 3$ since $|x - 3| = 0$, $x - 3 = 0$, $x = 3$
Hole: None

Solution to Exercise 9 (p. 2)
Vertical asymptotes: $x = -4$ and $x = 4$ since $|x| - 4 = 0$, $|x| = 4$, $x = -4$ and $x = 4$
Hole: None

Solution to Exercise 10 (p. 2)
Vertical Asymptote: $x = -3$
Hole: (3, $\frac{5}{8}$) since $\frac{3(x^2-x-6)}{4(x^2-9)} = \frac{3((x-3)(x+2))}{4((x+3)(x-3))} = \frac{3(x+2)}{4(x+3)}$, (x-3) was cancelled, so the hole is at x=3. To find the y-coordinate, plug 3 into the reduced equation: $\frac{3(3+2)}{4(3+3)} = \frac{3\times5}{4\times6} = \frac{15}{24} = \frac{5}{8}$

Solution to Exercise 11 (p. 2)
$\frac{-2(x^2-4)}{3(x^2+4x+4)} = \frac{-2(x+2)(x-2)}{3(x+2)^2} = \frac{-2(x-2)}{3(x+2)}$
Vertical Asymptote: $x = -2$
Hole: None since the vertical asymptote takes care of the hole.

Solution to Exercise 12 (p. 2)
Vertical Asymptote: None
Hole: (-2,-4) since $\frac{x^2-4}{x+2} = \frac{(x+2)(x-2)}{x+2} = x - 2$, (x+2) was cancelled, so the hole is at x = -2. To find the y-coordinate, plug -2 into the reduced equation: $-2 - 2 = -4$

Solution to Exercise 13 (p. 2)
Vertical Asymptotes: None
Holes: (3,3), (0,0) since $\frac{x^2(x-3)}{x^2-3x} = \frac{x^2(x-3)}{x(x-3)} = x$, x and (x-3) were cancelled, so the holes are at x=0 and x=3. To find the y-coordinate, plug 0 and 3 into the reduced equation: 0, 3

Solution to Exercise 14 (p. 2)
Vertical Asymptote: None

Hole: (1,3) since $\frac{x^3-1}{x-1} = \frac{(x-1)\left(x^2+x+1\right)}{x-1} = x^2 + x + 1$, (x-1) was cancelled, so the hole is at x=1. To find the y-coordinate, plug 1 into the reduced equation: $1^2 + 1 + 1 = 3$

Solution to Exercise 15 (p. 2)

$\frac{2x^2-3x-5}{x^2-1} = \frac{(2(x-5))(x+1)}{(x+1)(x-1)} = \frac{2(x-5)}{x-1}$

Vertical asymptote: $x = 1$ since $x - 1 = 0$

Hole: (-1, $\frac{7}{2}$) Since (x+1) was cancelled, the hole is at x= -1. To find the y-coordinate, plug -1 into the reduced equation: $\frac{2(-1-5)}{-1-1} = \frac{7}{2}$