M14 - Evaluation of the DTFT by the DFT*

C. Sidney Burrus

Abstract

Samples of the DTFT can be calculated using the DFT

1 Evaluation of the DTFT by the DFT

If the DTFT of a finite sequence is taken, the result is a continuous function of ω. If the DFT of the same sequence is taken, the results are N evenly spaced samples of the DTFT. In other words, the DTFT of a finite signal can be evaluated at N points with the DFT.

$$X(\omega) = DTFT\{x(n)\} = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\omega n} \tag{1}$$

and because of the finite length

$$X(\omega) = \sum_{n=0}^{N-1} x(n)\, e^{-j\omega n}. \tag{2}$$

If we evaluate ω at N equally space points, this becomes

$$X\left(\frac{2\pi}{N}k\right) = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi}{N}kn} \tag{3}$$

which is the DFT of $x(n)$. By adding zeros to the end of $x(n)$ and taking a longer DFT, any density of points can be evaluated. This is useful in interpolation and in plotting the spectrum of a finite length signal. This is discussed further in Chapter .

There is an interesting variation of the Parseval's theorem for the DTFT of a finite length-N signal. If $x(n) \neq 0$ for $0 \geq n \geq N-1$, and if $L \geq N$, then

$$\sum_{n=0}^{N-1} |x(n)|^2 = \frac{1}{L}\sum_{k=0}^{L-1} |X(2\pi k/L)|^2 = \frac{1}{\pi}\int_0^{\pi} |X(\omega)|^2 d\omega. \tag{4}$$

The second term in ((4)) says the Riemann sum is equal to its limit in this case.

*Version 1.1: Sep 17, 2006 12:27 pm GMT-5

†http://creativecommons.org/licenses/by/2.0/