m16 - Properties of the Z-Transform*

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Abstract

The properties of the z transform are similar to those of the DTFT.

1 Properties

The properties of the ZT are similar to those for the DTFT and DFT and are important in the analysis and interpretation of long signals and in the analysis and description of discrete-time systems. The main properties are given here using the notation that the ZT of a complex sequence $x(n)$ is $\mathscr{Z}\{x(n)\} = X(z)$.

1. Linear Operator: $\mathscr{Z}\{x+y\} = \mathscr{Z}\{x\} + \mathscr{Z}\{y\}$
2. Relationship of ZT to DTFT: $\mathscr{Z}\{x\}|_{z=e^{j\omega}} = \mathscr{DTFT}\{x\}$
3. Periodic Spectrum: $X\left(e^{j\omega}\right) = X\left(e^{j\omega+2\pi}\right)$
4. Properties of Even and Odd Parts: $x(n) = u(n) + jv(n)$ and $X\left(e^{j\omega}\right) = A\left(e^{j\omega}\right) + jB\left(e^{j\omega}\right)$

u	v	A	B
even	0	*even*	0
odd	0	0	*odd*
0	*even*	0	*even*
0	*odd*	*odd*	0

(1)

5. Convolution: If discrete non-cyclic convolution is defined by$y(n) = h(n)*x(n) = \sum_{m=-\infty}^{\infty} h(n-m)\,x(m) = \sum_{k=-\infty}^{\infty} h(k)\,x(n-k)$then $\mathscr{Z}\{h(n)*x(n)\} = \mathscr{Z}\{h(n)\}\mathscr{Z}\{x(n)\}$
6. Shift: $\mathscr{Z}\{x(n+M)\} = z^M X(z)$
7. Shift (unilateral): $\mathscr{Z}\{x(n+m)\} = z^m X(z) - z^m x(0) - z^{m-1}x(1) - \{\backslash\}\,\text{dots} - zx(m-1)$
8. Shift (unilateral): $\mathscr{Z}\{x(n-m)\} = z^{-m} X(z) - z^{-m+1}x(-1) - \{\backslash\}\,\text{dots} - x(-m)$
9. Modulate: $\mathscr{Z}\{x(n)\,a^n\} = X(z/a)$
10. Time mult.: $\mathscr{Z}\{n^m x(n)\} = (-z)^m \frac{d^m X(z)}{dz^m}$
11. Evaluation: The ZT can be evaluated on the unit circle in the z-plane by taking the DTFT of $x(n)$ and if the signal is finite in length, this can be evaluated at sample points by the DFT.

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