Non – uniform circular motion(application)*

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Abstract

Solving problems is an essential part of the understanding process.

Questions and their answers are presented here in the module text format as if it were an extension of the treatment of the topic. The idea is to provide a verbose explanation, detailing the application of theory. Solution presented is, therefore, treated as the part of the understanding process – not merely a Q/A session. The emphasis is to enforce ideas and concepts, which can not be completely absorbed unless they are put to real time situation.

1 Hints on problem solving

1: Calculation of acceleration as time rate of change of speed gives tangential acceleration.

2: Calculation of acceleration as time rate of change of velocity gives total acceleration.

3: Tangential acceleration is component of total acceleration along the direction of velocity. Centripetal acceleration is component of acceleration along the radial direction.

4: We exchange between linear and angular quantities by using radius of circle, "r", as multiplication factor. It is helpful to think that linear quantities are bigger than angular quantities. As such, we need to multiply angular quantity by "r" to get corresponding linear quantities and divide a linear quantity by "r" to get corresponding angular quantity.

2 Representative problems and their solutions

We discuss problems, which highlight certain aspects of the study leading to non-uniform circular motion. The questions are categorized in terms of the characterizing features of the subject matter :

- Velocity
- Average total acceleration
- Total acceleration

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3 Velocity

Example 1

Problem : A particle, tied to a string, starts moving along a horizontal circle of diameter 2 m, with zero angular velocity and a tangential acceleration given by " $4t$ ". If the string breaks off at t = 5 s, then find the speed of the particle with which it flies off the circular path.

Solution : Here, tangential acceleration of the particle at time "t" is given as :

$$a_T = 4t$$

We note here that an expression of tangential acceleration (an higher attribute) is given and we are required to find lower order attribute i.e. linear speed. In order to find linear speed, we need to integrate the acceleration function :

$$a_T = \frac{v}{t} = 4t$$

$$\Rightarrow v = 4tt$$

Integrating with appropriate limits, we have :

$$\Rightarrow \int v = \int 4tt = 4 \int tt$$

$$\Rightarrow v_f - v_i = 4 \left[\frac{t^2}{2} \right]_0^5 = \frac{4X5^2}{2} = 50 \quad m/s$$

$$\Rightarrow v_f - 0 = 50$$

$$\Rightarrow v_f = 50 \quad m/s$$

4 Average total acceleration

Example 2

Problem : A particle is executing circular motion. The velocity of the particle changes from $(0.1\mathbf{i} + 0.2\mathbf{j})$ m/s to $(0.5\mathbf{i} + 0.5\mathbf{j})$ m/s in a period of 1 second. Find the magnitude of average total acceleration.

Solution : The average total acceleration is :

$$\mathbf{a} = \frac{\Delta \mathbf{v}}{\Delta t} = \frac{(0.5\mathbf{i} + 0.5\mathbf{j}) - (0.1\mathbf{i} + 0.2\mathbf{j})}{1}$$

$$\mathbf{a} = (0.4\mathbf{i} + 0.3\mathbf{j})$$

The magnitude of acceleration is :

$$a = \sqrt{(0.4^2 + 0.3^2)} = \sqrt{0.25} = 0.5\, m/s$$

Example 3

Problem : A particle starting with a speed “v” completes half circle in time “t” such that its speed at the end is again “v”. Find the magnitude of average total acceleration.

Solution : Average total acceleration is equal to the ratio of change in velocity and time interval.

$$a_{\text{avg}} = \frac{v_2 - v_1}{\Delta t}$$

From the figure and as given in the question, it is clear that the velocity of the particle has same magnitude but opposite directions.

Circular motion

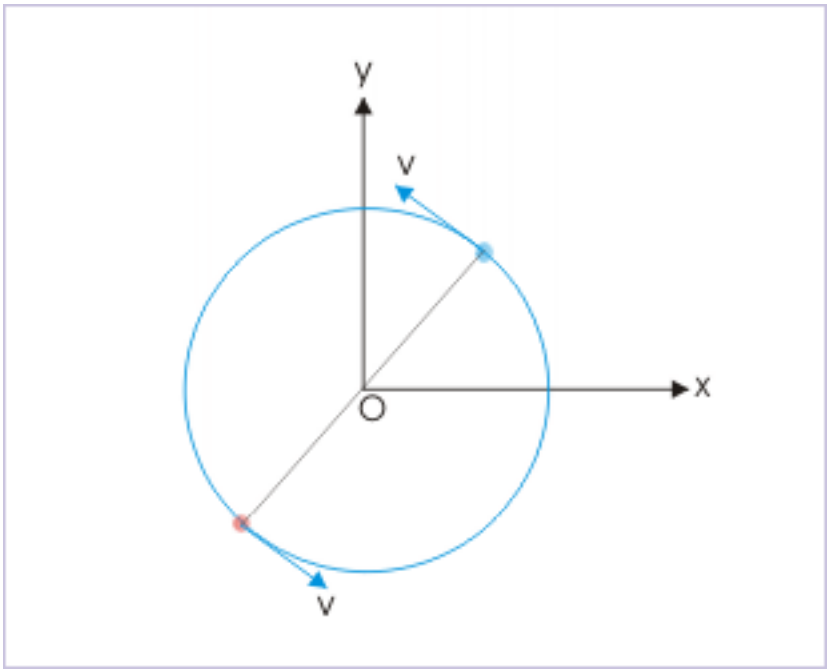

Figure 1: The speeds of the particle are same at two positions.

$$v_1 = v$$

$$v_2 = -v$$

Putting in the expression of average total acceleration, we have :

$$a_{\text{avg}} = \frac{v_2 - v_1}{\Delta t} = \frac{-v - v}{t}$$

$$a_{\text{avg}} = \frac{-2v}{t}$$

The magnitude of the average acceleration is :

$$\Rightarrow a_{\text{avg}} = \frac{2v}{t}$$

5 Total acceleration

Example 4

Problem : The angular position of a particle (in radian), on circular path of radius 0.5 m, is given by :

$$\theta = -0.2t^2 - 0.04$$

At t = 1 s, find (i) angular velocity (ii) linear speed (iii) angular acceleration (iv) magnitude of tangential acceleration (v) magnitude of centripetal acceleration and (vi) magnitude of total acceleration.

Solution : Angular velocity is :

$$\omega = \frac{\theta}{t} = \frac{}{t}\left(-0.2t^2 - 0.04\right) = -0.2X2t = -0.4t$$

The angular speed, therefore, is dependent as it is a function in "t". At t = 1 s,

$$\omega = -0.4 \quad \text{rad}/s$$

The magnitude of linear velocity, at t = 1 s, is :

$$v = \omega r = 0.4X0.5 = 0.2 \quad m/s$$

Angular acceleration is :

$$\alpha = \frac{\omega}{t} = \frac{}{t}\left(-0.4t\right) = -0.4 \quad \text{rad}/s^2$$

Clearly, angular acceleration is constant and is independent of time.
The magnitude of tangential acceleration is :

$$\Rightarrow a_T = \alpha r = 0.4X0.5 = 0.2 \quad m/s^2$$

Tangential acceleration is also constant and is independent of time.
The magnitude of centripetal acceleration is :

$$\Rightarrow a_R = \omega v = 0.4tX0.2t = 0.08t^2$$

At t = 1 s,

$$\Rightarrow a_R = 0.08 \quad m/s^2$$

The magnitude of total acceleration, at t =1 s, is :

$$a = \sqrt{(a_T^2 + a_R^2)}$$

$$a = \sqrt{\{(0.2)^2 + (0.08)^2\}} = 0.215 \quad m/s^2$$

Example 5

Problem : The speed (m/s) of a particle, along a circle of radius 4 m, is a function in time, "t" as :

$$v = t^2$$

Find the total acceleration of the particle at time, t = 2 s.

Solution : The tangential acceleration of the particle is obtained by differentiating the speed function with respect to time,

$$a_T = \frac{v}{t} = \frac{}{t}\left(t^2\right) = 2t$$

The tangential acceleration at time, t = 2 s, therefore, is :

$$\Rightarrow a_T = 2X2 = 4 \quad m/s^2$$

The radial acceleration of the particle is given as :

$$a_R = \frac{v^2}{r}$$

In order to evaluate this expression, we need to know the velocity at the given time, t = 2 s :

$$\Rightarrow v = t^2 = 2^2 = 4 \quad m/s$$

Putting in the expression of radial acceleration, we have :

$$\Rightarrow a_R = \frac{v^2}{r} = \frac{4^2}{4} = 4 \quad m/s^2$$

The total acceleration of the particle is :

$$a = \sqrt{(a_T^2 + a_R^2)} = \sqrt{(4^2 + 4^2)} = 4\sqrt{2} \quad m/s^2$$