Conservation of energy (application)*

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Abstract

Solving problems is an essential part of the understanding process.

Questions and their answers are presented here in the module text format as if it were an extension of the treatment of the topic. The idea is to provide a verbose explanation, detailing the application of theory. Solution presented is, therefore, treated as the part of the understanding process – not merely a Q/A session. The emphasis is to enforce ideas and concepts, which can not be completely absorbed unless they are put to real time situation.

1 Representative problems and their solutions

We discuss problems, which highlight certain aspects of the study leading to the conservation of energy. The questions are categorized in terms of the characterizing features of the subject matter :

- Particle
- Projectile
- Pulley - block system
- Incline – block - spring system
- Vertical circular motion

2 Particle

Problem 1: The potential energy of a particle of mass "m" is given by " $\frac{1}{2}Ax^2$ ". If non-conservative force is not involved and total mechanical energy is "E", then find its speed at "x".

Solution : External force and non-conservative force are absent here. Hence, we can apply conservation of mechanical energy. Total mechanical energy of the particle is constant,

$$E = U + K$$

$$\Rightarrow K = E - U$$

*Version 1.4: Sep 3, 2009 6:17 am -0500

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$$\Rightarrow K = E - \frac{1}{2}Ax^2$$

$$\Rightarrow \frac{1}{2}mv^2 = E - \frac{1}{2}Ax^2$$

$$\Rightarrow v = \sqrt{\{\frac{(2E - Ax^2)}{m}\}}$$

3 Projectile

Problem 2: A ball of 10 gm is fired from a spring gun of spring constant 500 N/m. The spring is initially compressed by 5 cm and is, then, released. Find the maximum horizontal distance (in meter) traveled by the bullet.

Solution : We are required to find maximum horizontal distance. The horizontal range of a projectile from the ground at an angle "θ" is given by :

Projectile

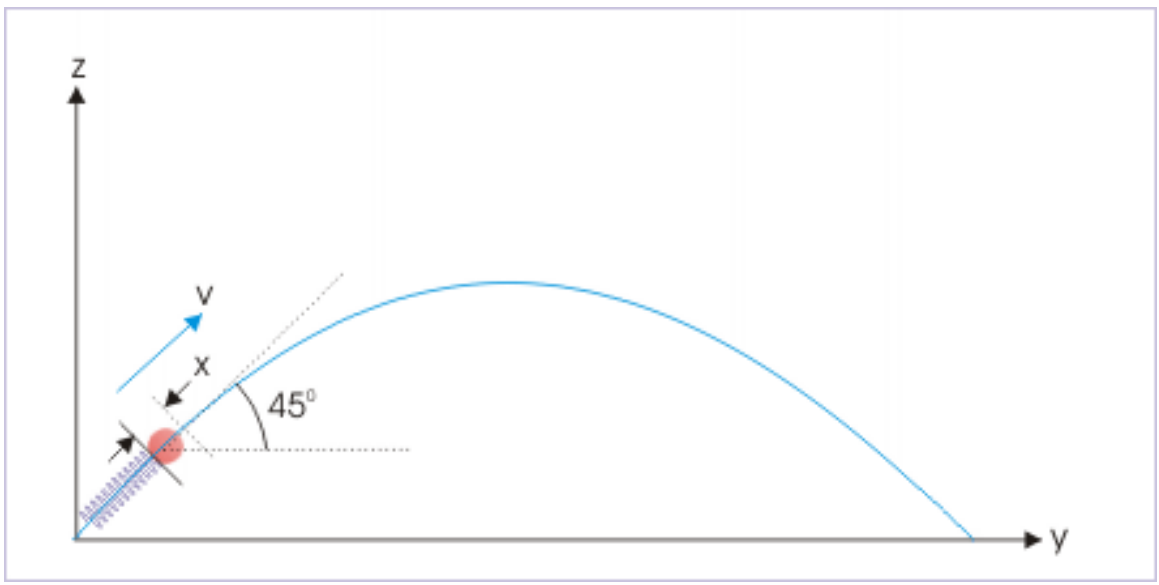

Figure 1: A ball is fired from spring gun.

$$R = \frac{v^2 \sin 2\theta}{g}$$

For maximum horizontal range, the angle of projection should be 45 °. The maximum range is :

$$\Rightarrow R = \frac{v^2}{g}$$

In order to find speed of projection, we shall employ conservation of mechanical energy. The gun and ball forms an isolated system. Initially, kinetic energy of the bullet is zero, whereas initial elastic potential energy of the spring is " $\frac{1}{2}kx^2$ ". Let "v" be the speed when ball breaks the contact with spring. Now, applying conservation of mechanical energy,

$$K_i + U_i = K_f + U_f$$

Here,

$$K_i = 0$$

$$U_i = \frac{1}{2}kx^2$$

$$K_f = \frac{1}{2}mv^2$$

$$U_f = 0$$

Thus,

$$\Rightarrow \frac{1}{2}kx^2 = \frac{1}{2}mv^2$$

$$\Rightarrow v^2 = \frac{kx^2}{m}$$

Putting this expression in the expression of horizontal range, the maximum horizontal range is :

$$\Rightarrow R = \frac{kx^2}{mg}$$

$$\Rightarrow R = \frac{500X(0.05)^2}{0.01X10} = \frac{500x0.0025}{0.1} = \frac{1.25}{0.1} = 12.5 \quad m$$

4 Pulley - block system

4.1

Problem 3: In the arrangement shown, the lighter block ascends a height of 2 m after being released. Find the speed of the blocks. Consider string to be mass-less and no friction at the contact.

Pulley - block system

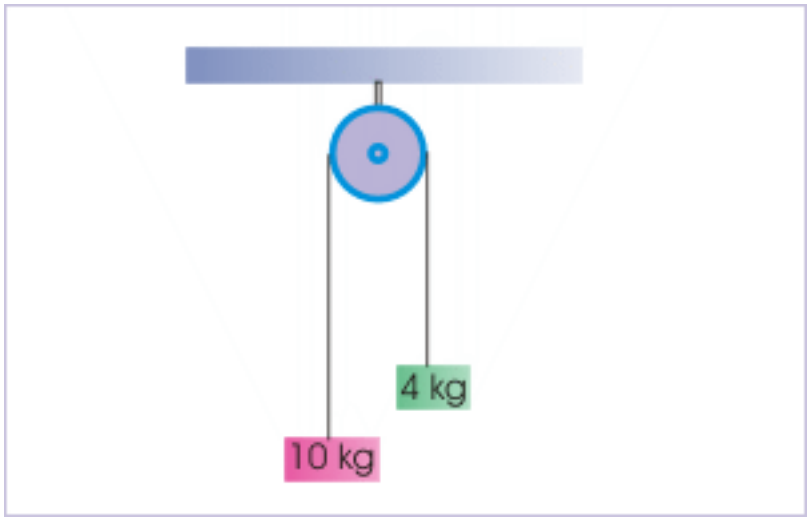

Figure 2: The blocks move vertically by 2m.

Solution : Since friction is absent, only gravity interacts with the elements of the system. We can consider arrangement as isolated system. No external force acts on the system. Remember that Earth is part of the system and gravitational force is internal to the system. Hence, the situation fulfills all conditions for applying conservation of mechanical energy.

Now, two blocks are constrained by a taut string. It means that both blocks move with same speed. Let us denote 4 kg and 10 kg blocks with subscript "1" and "2". Note that data is given for the change in height. Hence, it is easier to determine change in the potential energy.

$$\Delta U = m_1 gh + m_2 gh = 4X10X2 - 10X10X2 = -120 \quad J$$

Now, the change in kinetic energy is given by :

$$\Delta K = \frac{1}{2} m_1 v^2 + \frac{1}{2} m_2 v^2$$

$$\Rightarrow \Delta K = \frac{1}{2} X4Xv^2 + \frac{1}{2} X10Xv^2$$

$$\Rightarrow \Delta K = 7v^2$$

Now applying conservation of mechanical energy, we have :

$$\Delta K + \Delta U = 0$$

$$\Rightarrow 7v^2 - 120 = 0$$

$$\Rightarrow v = 4.14 \quad m/s$$

4.2

Problem 4: The block of mass 10 kg is released in the arrangement consisting of various elements as shown in the figure. If spring constant is 50 N/m, then find the speed of 10 kg block, when block of 5 kg is about to leave the ground. (Consider g = 10 m/s^2)

Pulley - block system

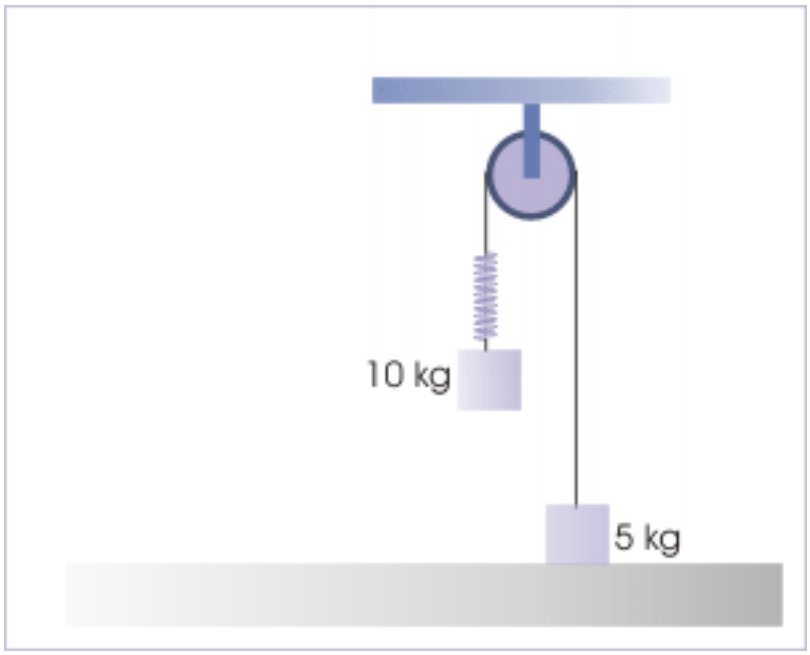

Figure 3: The block resting on the floor is about to be lifted.

Solution : Before the block of 10 kg is released, the kinetic energy of the system is zero. We are required to find the speed of the released block, when the block in contact with horizontal surface is about to be lifted. It means that there is no change in kinetic or potential energy for the block of 5 kg.

In order to assess the energy state of the block of 10 kg, let us analyze the process. As the block of 10 kg moves down, it stretches the spring. As such, it acquires the speed (kinetic energy), while spring acquires elastic potential energy. However, at the same time, the gravitational potential energy of the block decreases as it comes down. Let the extension in the spring be "x" and let the speed of the mass of 10 kg be "v", when block of 5 kg is about to be lifted.

Considering the system isolated, the change in kinetic energy is :

$$\Delta K = \frac{1}{2}mv^2 = \frac{1}{2}X10Xv^2 = 5v^2$$

The change in potential energy is :

$$\Delta U = \frac{1}{2}kx^2 - mgx$$

We need to know "x" to evaluate the expression of potential energy. When the block of 5kg is about to be lifted, then force across the spring is equal to the difference of weights. Applying Hook'e law,

$$Kx = (10 - 5)\, g = 5X10 = 50$$

$$\Rightarrow x = \frac{50}{50} = 1 \quad m$$

Now, applying conservation of energy,

$$\Delta U + \Delta K = 0$$

$$\Rightarrow \frac{1}{2}kx^2 - mgx + 5v^2 = 0$$

$$\Rightarrow v = \frac{10X10X1 - \frac{1}{2}X50X1^2}{5}$$

$$\Rightarrow v = 15 \quad m/s$$

5 Incline – block - spring system

Problem 5: We are designing a delivery ramp for crates containing exercise equipment. The 1470-N crates will move at 1.8 m/s at the top of a ramp that slopes downward at 22 degrees. The ramp exerts a 550-N kinetic friction force on each crate, and the maximum static friction also has this value. Each crate will compress a spring at the bottom of the ramp and will come to rest after traveling a total distance of 8.0 meters along the ramp. Once stopped, a crate must not rebound back up the ramp. Calculate the maximum force constant of the spring which will meet the design criteria. Take sin22 $^\circ$ = 0.374 and g = 10 m/s^2.

NOTE: This problem is adapted from the question mailed by a student.

Incline – block - spring system

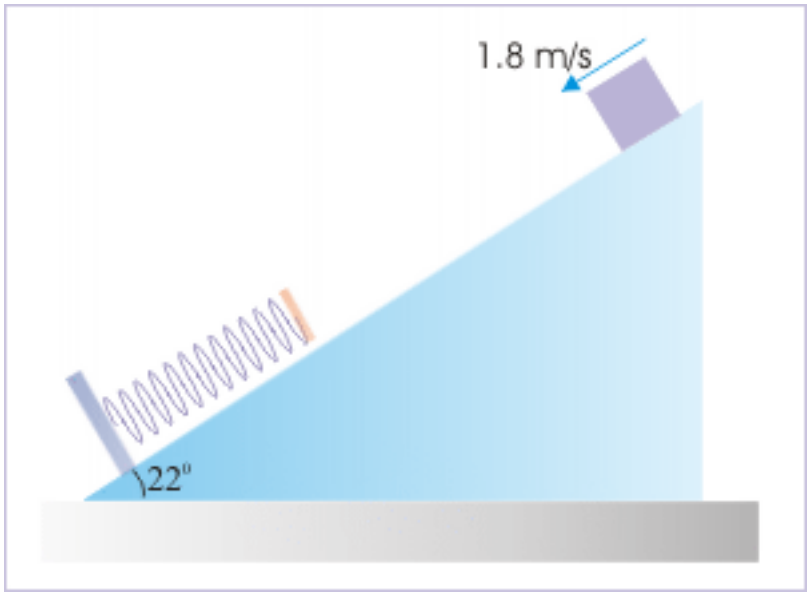

Figure 4: The block is moving with an initial speed down the incline.

Solution : Here the critical part of the question is to understand that the force required to return the crate up is equal to the sum of the component of its weight down the incline and the maximum static friction so that block can be initiated up the incline. Hence, spring force for this condition is :

Incline – block - spring system

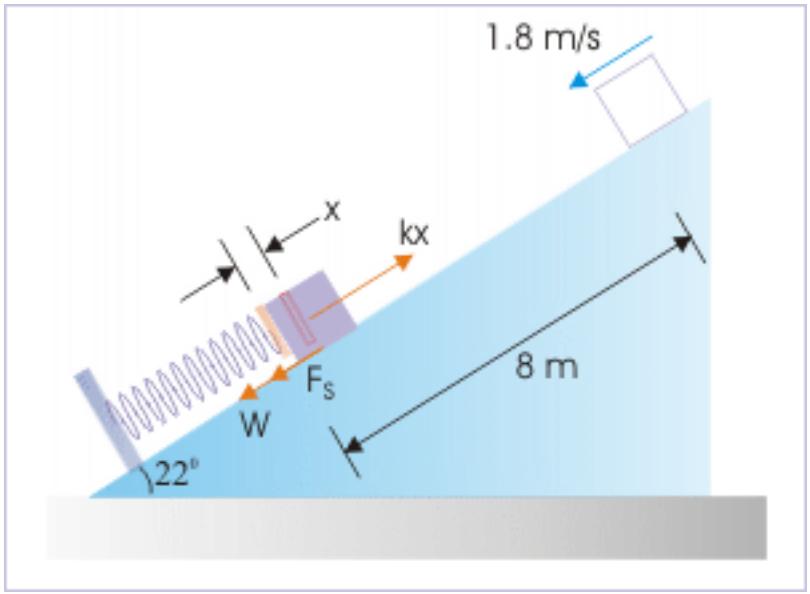

Figure 5: Forces on the crate.

$$F_S = mg\sin\theta + F_K = mg\sin 22^0 + 550 = 1470X0.374 + 550 = 1100 \quad N$$

Let "x" be the compression in the spring. Then, applying Hooke' law, we get the compression in the spring as :

$$F_S = Kx = 1100$$

The system here is an isolated system with both conservative and non-conservative force. Hence,

$$W_F = \Delta K + \Delta U$$

Here,

$$\Rightarrow W_F = -550x8 = -4400 \quad J$$

$$\Rightarrow \Delta K = K_f - K_i = 0 - \frac{1}{2}X147X1.82 = -238.14 \quad J$$

$$\Rightarrow \Delta U = \frac{1}{2}kx^2 - mgh = \frac{1}{2}kx^2 - 1470X8X\sin 22^0$$

$$\Rightarrow \Delta U = \frac{1}{2}kx^2 - 1470X8X\sin 22^0 = \frac{1}{2}kx^2 - 4398$$

Putting values in the equation, we have :

$$\Rightarrow W_F = \Delta K + \Delta U$$

$$\Rightarrow -4400 = -238 + \frac{1}{2}kx^2 - 4398$$

$$\Rightarrow \frac{1}{2}kx^2 = 238$$

$$\Rightarrow kx^2 = 476$$

Combining with Hooke's equation,

$$\Rightarrow K = \frac{476K^2}{1100^2}$$

$$\Rightarrow K = \frac{1100^2}{476} = 2542 \quad N/m$$

NOTE: We can see that a spring having spring constant less than this value will also not return the crate. Hence, this spring constant represents the maximum value of the spring constant, which meets the required design criterion.

6 Vertical circular motion

Problem 5: An object inside a spherical shell is projected horizontally at the lowest point such that it just completes the circular motion in a vertical plane. If the radius of the shell is 2 m, then find the acceleration of the object when its velocity is vertical.

Vertical circular motion

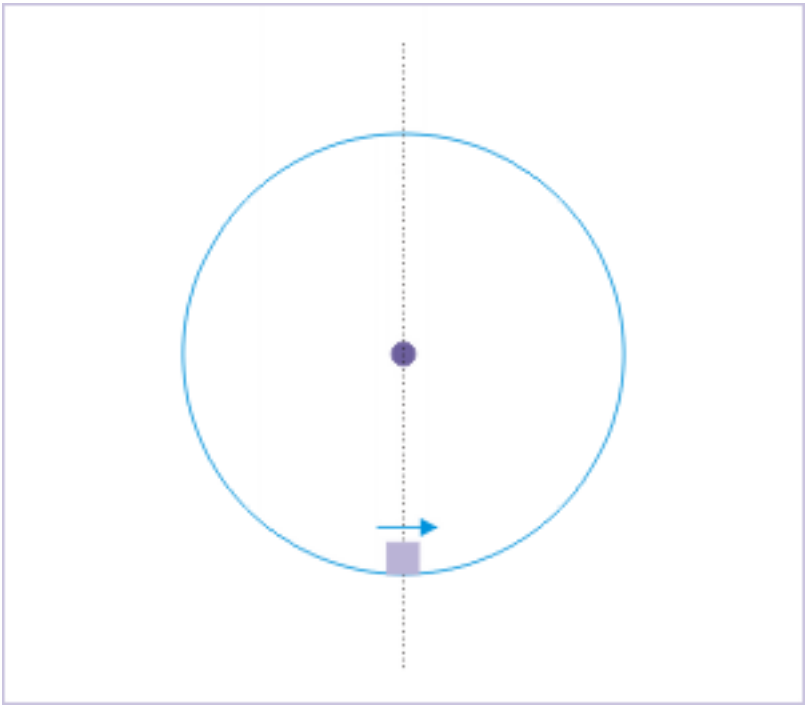

Figure 6: An object inside a spherical shell is projected horizontally at the lowest point.

Solution : The object attains velocity in vertical direction, when it reaches half of the total height. The acceleration of the object is resultant of tangential and centripetal accelerations at that point :

Vertical circular motion

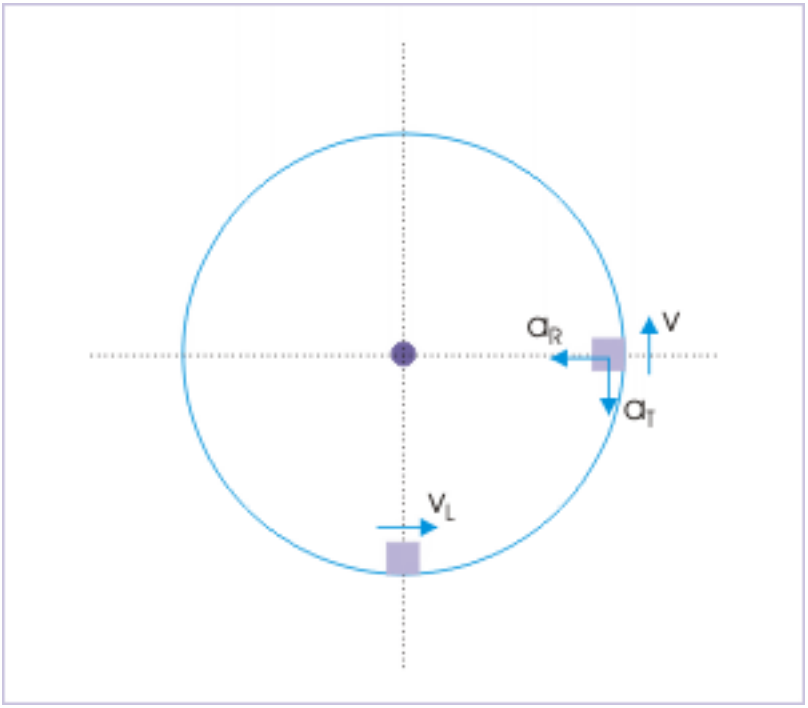

Figure 7: The acceleration of the object is resultant of tangential and centripetal accelerations.

$$a = \sqrt{a_T^2 + a_R^2}$$

Here,

$$a_T = g$$

and

$$a_r = \frac{v^2}{r}$$

We need to know the speed of the object when it has reached half the height. Now, the minimum speed at the lowest point to enable to complete the revolution is :

$$v_L = \sqrt{5gr}$$

From conservation of energy at lowest and middle levels,

$$K_L + U_L = K_M + U_M$$

Considering zero reference potential at the lowest point,

$$U_L = 0$$

$$U_M = mgr$$

Putting these values in energy equation, we have :

$$\Rightarrow \frac{1}{2}mv^2 = \frac{1}{2}m\left(\sqrt{5gr}\right)^2 - mgr$$

$$\Rightarrow v^2 = \left(\sqrt{5gr}\right)^2 - 2gr = 3gr$$

and

$$\Rightarrow a_r = \frac{v^2}{r} = \frac{3gr}{r} = 3g$$

Thus,

$$\Rightarrow a = \sqrt{\{g^2 + (3g)^2\}} = \sqrt{10}g = 10\sqrt{10} \quad m/s^2$$

Note : It is interesting to note that acceleration is independent of the radius of the shell.