Working with moving pulleys (application)*

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Abstract

Solving problems is an essential part of the understanding process.

Questions and their answers are presented here in the module text format as if it were an extension of the treatment of the topic. The idea is to provide a verbose explanation, detailing the application of theory. Solution presented is, therefore, treated as the part of the understanding process – not merely a Q/A session. The emphasis is to enforce ideas and concepts, which can not be completely absorbed unless they are put to real time situation.

1 Representative problems and their solutions

We discuss problems, which highlight certain aspects of the study leading to working with pulleys. The questions are categorized in terms of the characterizing features of the subject matter :

- Block and pulley system
- Incline and pulley system

2 Block and pulley system

2.1

Problem 1 : In the arrangement, find the mass "M" such that block of mass 1 kg moves with uniform velocity. Neglect mass of the pulleys and the string. Also neglect friction between the surfaces.

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Block and pulley system

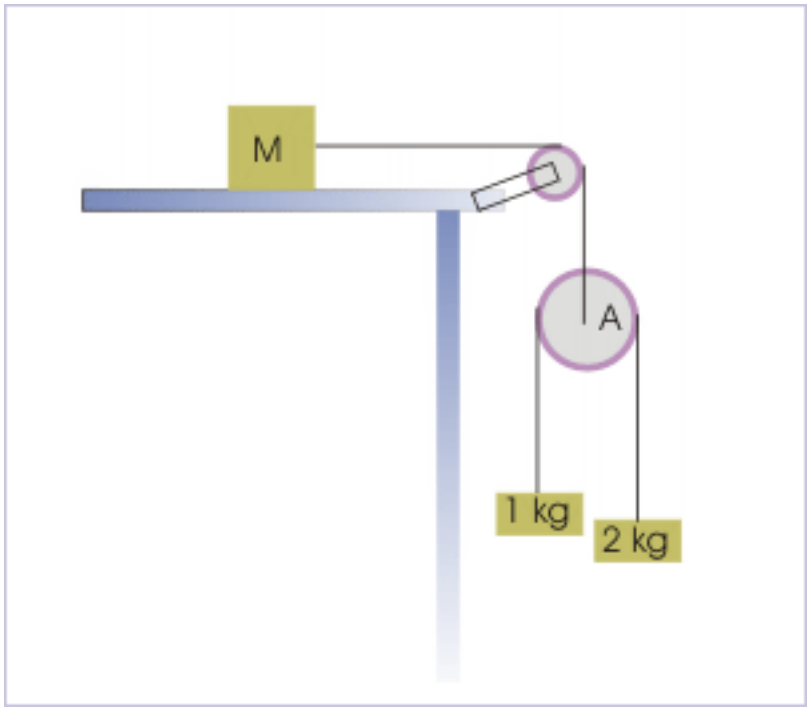

Figure 1: The block on the table is attached to a pulley system.

Solution : The block of mass 1 kg moves with constant velocity with respect to ground. It means that there is no net external force on the block. Let us denote block of mass 1 kg and 2 kg by the subscripts "1" and "2" respectively.

Now, forces on the block of mass 1 kg, are (i) its weight, " $m_1 g$ ", acting downward and (i) tension in the string, say " T_1 ", acting upward. The two forces form a balanced force system as block moves with uniform velocity. Hence,

Free body diagrams

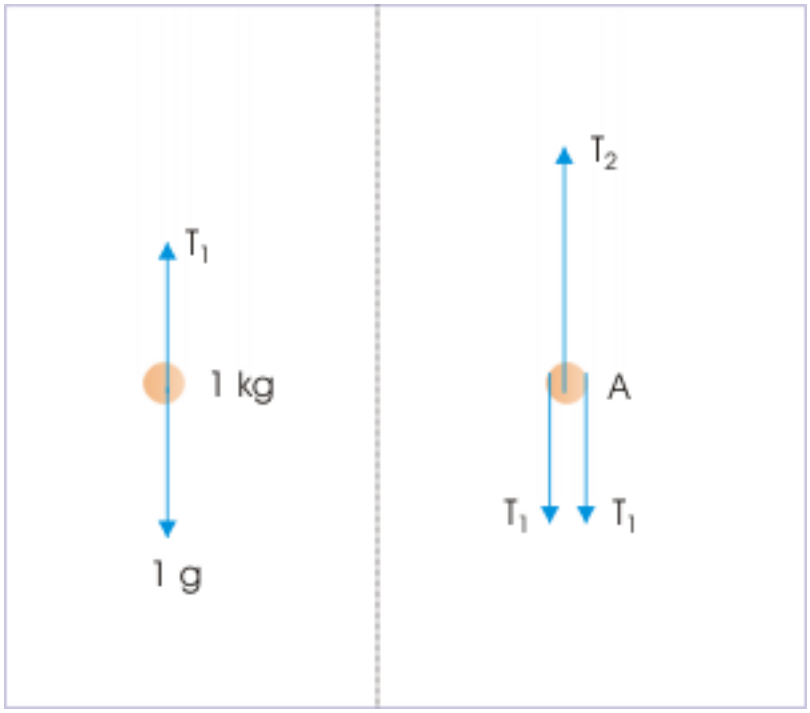

Figure 2: FBD of the block of 1 kg and moving pulley

$$\Rightarrow T_1 = m_1 g = 1X10 = 10 \quad N$$

From the force analysis of the moving mass-less pulley, "A",

$$T_2 = 2T_1$$

Combining two equations,

$$\Rightarrow T_2 = 2X10 = 20 \quad N$$

Tension is the only force on the block, which is placed on the horizontal surface of the table. From the force analysis of the block on the horizontal surface (see FBD of block in the figure below), we have :

$$T_2 = Ma$$

$$20 = MXa$$

$$a = \frac{20}{M}$$

This is also the acceleration of moving pulley as it is connected to the block on the table with a string. If we consider upward direction as positive, then acceleration of the moving pulley is :

Free body diagram

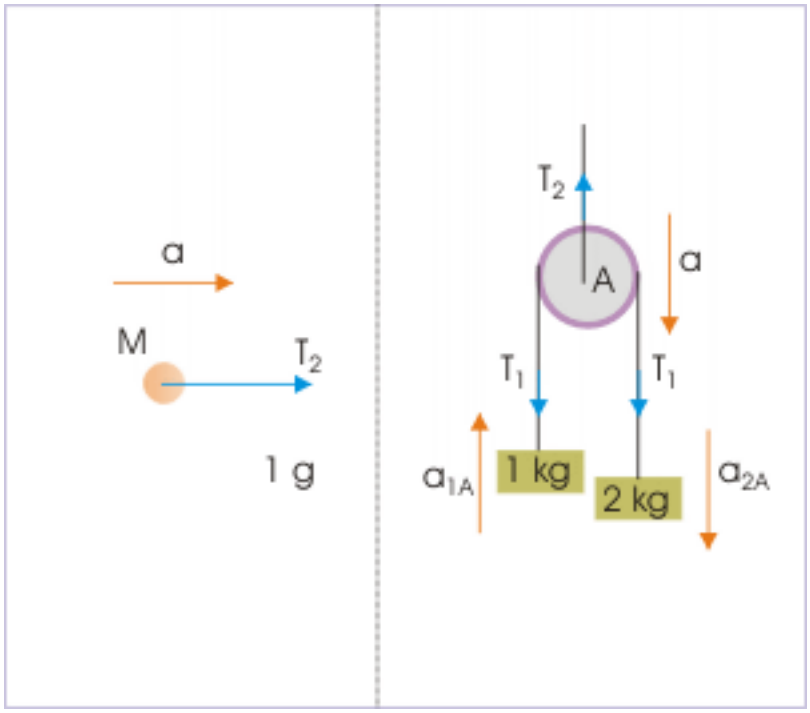

Figure 3: FBD of the block on the table and forces on the moving pulley system

$$\Rightarrow a_A = -\frac{20}{M} \quad \text{downward}$$

From the expression of acceleration of moving pulley, it is clear that we should get an equation involving acceleration of moving pulley in order to ultimately determine the value of "M" as required. According to question, the block of mass “1 kg” travels with zero acceleration (constant velocity) with reference to ground. The force analysis of this mass, therefore, does not involve acceleration of the moving pulley. Thus, we shall strive to find the acceleration of block of "2 kg" with respect to ground as its expression will involve acceleration of the moving pulley. Since block of mass “1 kg” travels with zero acceleration (constant velocity) with reference to ground, its acceleration with respect to moving pulley is :

We shall, however, take advantage of the fact that acceleration of block of 1 kg has zero acceleration with respect to ground reference. Its acceleration with respect to moving pulley is :

$$a_{1A} = a_1 - a_A$$

$$\Rightarrow a_{1A} = 0 - \left(-\frac{20}{M}\right) = \frac{20}{M} \text{upward}$$

This result is physically verifiable. Since pulley has downward acceleration, the block should have upward acceleration of the same magnitude such that block acceleration with respect to ground is zero. The blocks

are connected by the same string that goes over the pulley. It means that relative accelerations of the blocks are equal but opposite in direction (see Working with moving pulleys [1]).

$$\Rightarrow a_{2A} = -a_{1A} = -\frac{20}{M}\text{downward}$$

The acceleration of mass " m_2 " with respect to ground can, now, be obtained as :

$$a_{2A} = a_2 - a_A$$

$$a_2 = a_{2A} + a_A = -\frac{20}{M} + \left(-\frac{20}{M}\right) = -\frac{40}{M}$$

From force analysis of block of mass 2 kg, we have :

$$-m_2 g + T_1 = -m_2 a_2$$

$$m_2 g - T_1 = m_2 a_2$$

$$\Rightarrow 2X10 - 10 = 2X\frac{40}{M}$$

$$\Rightarrow 10M = 80$$

$$\Rightarrow M = 8kg$$

2.2

Problem 2 : For the arrangement as shown in the figure, find the acceleration of block on the horizontal floor. Consider all surfaces "friction-less" and strings and pulleys as "mass-less" for the force analysis.

[1] "Working with moving pulleys" <http://cnx.org/content/m14782/latest/>

Block and pulley system

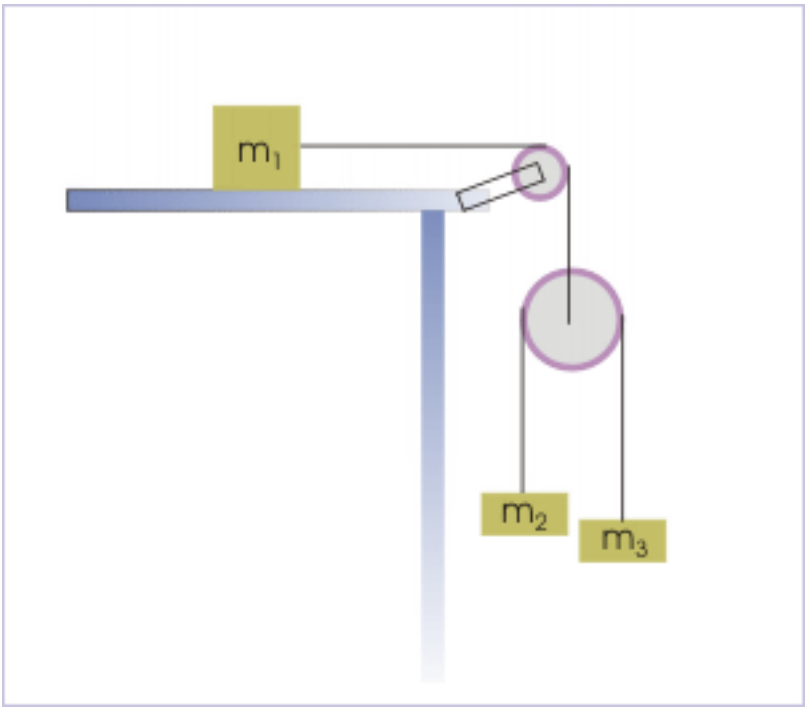

Figure 4: The block on the table is attached to a pulley system.

Solution : If we look at the motions of blocks from the reference of moving pulley, then the situation is that of motion of two blocks and a static pulley. Since the blocks are connected by the same string, the accelerations of the blocks with respect to pulley are equal in magnitude but opposite in direction.

Block and pulley system

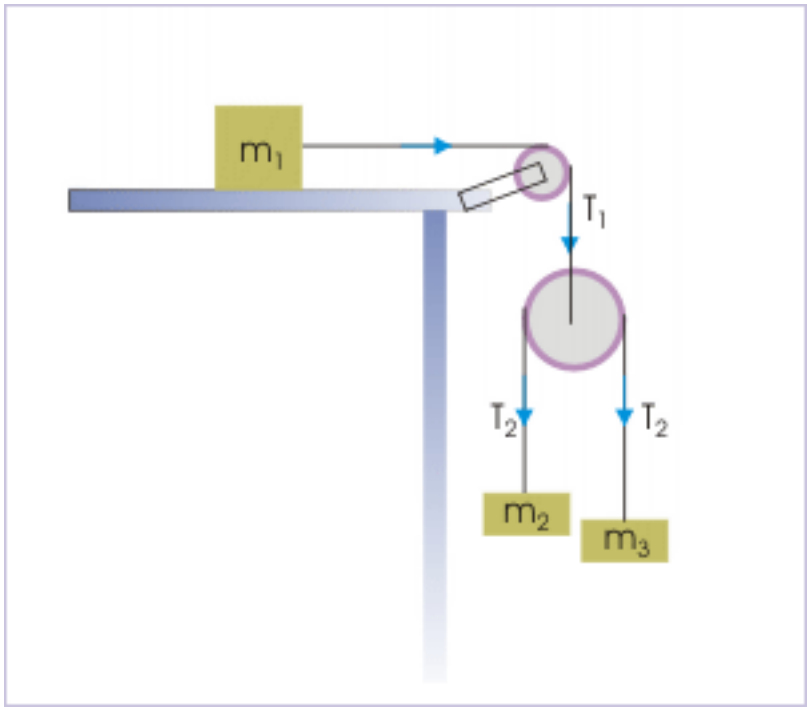

Figure 5: Tensions in the string are shown.

Let the acceleration of the block of mass, " m_1 ", placed on the horizon, be " a_1 ". Since block on the horizontal surface and the moving pulley are connected by a single piece of string, the acceleration of the moving pulley is same. As such, acceleration of the moving pulley is " a_1 ". Now, let the relative accelerations of the blocks with respect to moving pulley be " a_{21} " and " a_{31}" respectively.

In order to carry out force analysis of the different elements of the system, we are required to express accelerations with respect to ground. This is an important point. Note that Newton's laws of motion is applicable to inertial frame of reference - not to an accelerated frame of reference like that of moving pulley. We can convert "accelerations with respect to accelerating pulley" to "accelerations with respect to ground" by using the concept of relative acceleration. Let block of mass " m_2 " moves down and the block of mass " m_3 " moves up with respect to the moving pulley. Also, let their accelerations with respect to ground be " a_2 " and " a_3 " respectively. Then,

$$a_{21} = a_2 - a_1$$

$$a_{31} = a_3 - a_1$$

But magnitudes of relative accelerations with respect to moving pulley are equal.

$$|a_{21}| = |a_{31}| = a_r$$

Then, considering downward direction as positive,

$$\Rightarrow a_2 = a_1 + a_{21} = a_1 - a_r$$

$$\Rightarrow a_3 = a_1 + a_{31} = a_1 + a_r$$

Thus, we see that two accelerations are now expressed in terms of one relative acceleration, " a_r ", only. The analysis of forces in the ground reference for different elements of the system are :

Free body diagrams

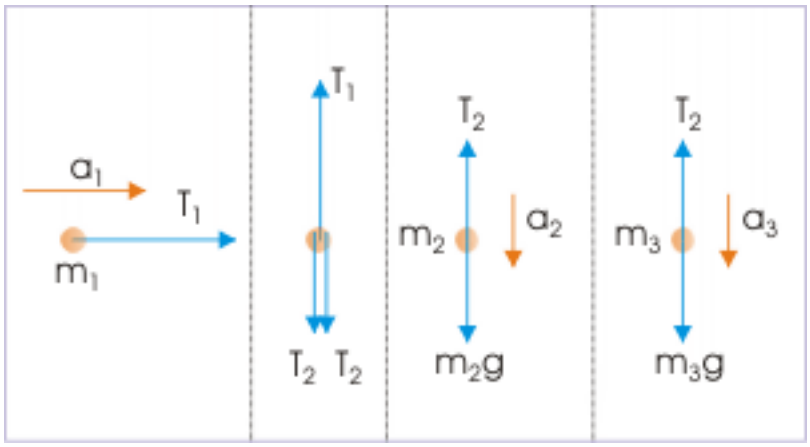

Figure 6: Free body diagrams of block on the table, moving pulley and two blocks are shown.

(i) The block of mass, " m_1 " :

$$T_1 = m_1 a_1$$

(ii) For pulley :

$$T_2 = \frac{T_1}{2}$$

Combining two equations,

$$T_2 = \frac{m_1 a_1}{2}$$

(iii) For block of mass, " m_2 " :

$$m_2 g - T_2 = m_2 a_2$$

$$\Rightarrow m_2 g - T_2 = m_2 \left(-a_r + a_1\right)$$

(iv) For block of mass, " m_3 " :

$$\Rightarrow m_3 g - T_2 = m_3 \left(a_r + a_1\right)$$

We need to eliminate " a_r " and solve two equations for " T_2 ". For this we multiply first by " m_3 " and second by " m_2 ". The new pair of equations are,

$$\Rightarrow m_2 m_3 g - m_3 T_2 = m_2 m_2 a_1 - m_2 m_3 a_r$$

$$\Rightarrow m_2 m_3 g - m_2 T_2 = m_2 m_3 a_1 + m_2 m_3 a_r$$

Adding, we have :

$$\Rightarrow 2 m_2 m_3 g - (m_2 + m_3) T_2 = 2 m_2 m_3 a_1$$

Substituting for " T_2 ", we have :

$$\Rightarrow 2 m_2 m_3 g - (m_2 + m_3) \frac{m_1 a_1}{2} = 2 m_2 m_3 a_1$$

$$a_1 \left[2 m_2 m_3 + (m_2 + m_3) \frac{m_1}{2} \right] = 2 m_2 m_3 g$$

$$\Rightarrow a_1 \{ 4 m_2 m_3 + (m_2 + m_3) m_1 \} = 4 m_2 m_3 g$$

$$\Rightarrow a_1 = \frac{4 m_2 m_3 g}{\{ 4 m_2 m_3 + (m_2 + m_3) m_1 \}}$$

3 Incline and pulley system

3.1

Problem 3 : Two blocks of mass "m" and "nm" are part of the arrangement involving pulley and incline as shown in the figure. If friction is negligible everywhere and mass of the pulleys and the strings are also negligible, then find the acceleration of the mass "nm".

Incline and pulley system

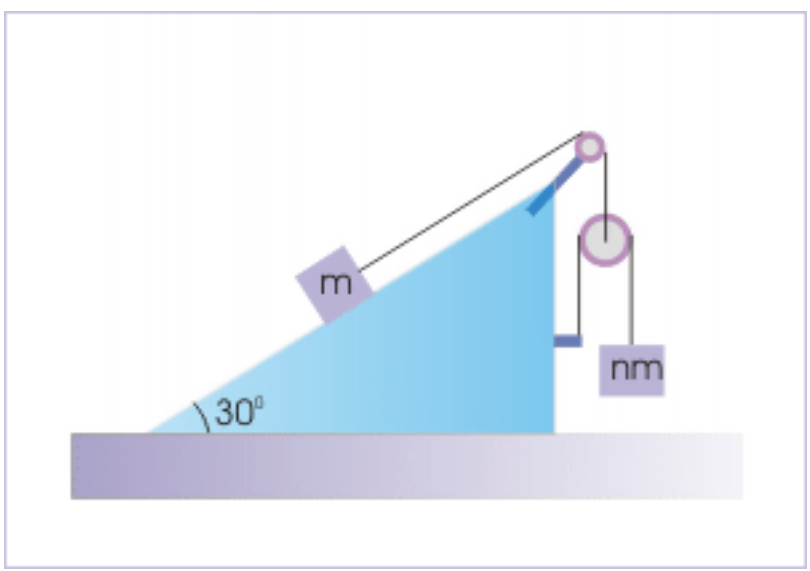

Figure 7: The block on incline has tendency to move down.

Solution : Here, we see that block of mass "m" is connected to the moving pulley by a single string. Hence, their accelerations are equal in magnitude. Let the acceleration of the block of mass "m" be " a_1 " and the acceleration of block of mass "nm" be " a_2 ". Similarly, let the tensions in the two strings be " T_1 " and " T_2 " respectively. The force diagrams of the elements of the system are shown here.

Incline and pulley system

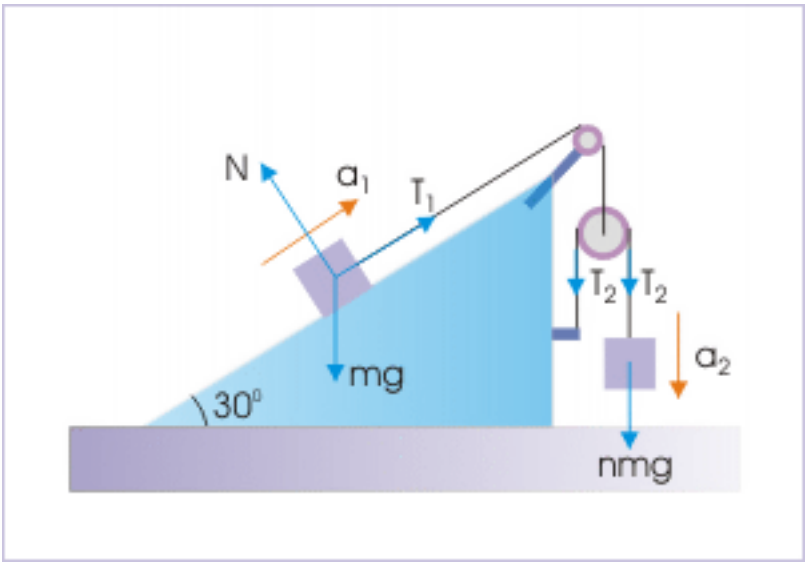

Figure 8: Forces on the elements of the system are shown.

The free body diagrams of block on the incline, moving pulley and hanging block are shown here :

Free body diagrams

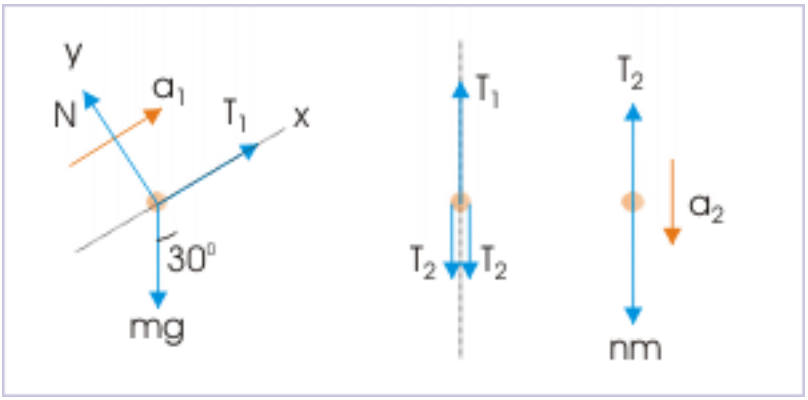

Figure 9: Free body diagrams of block on incline, pulley and block hanging from pulley are shown.

$$T_1 - mg\sin 30^0 = ma_1$$

$$T_1 = 2T_2$$

$$nmg - T_2 = nma_2$$

Now, we need to establish the relation between accelerations of pulley and block of mass "nm" by applying constraint analysis. For this we consider a horizontal reference passing through the fixed end of the string. Using length of string as constraint, we have :

Incline and pulley system

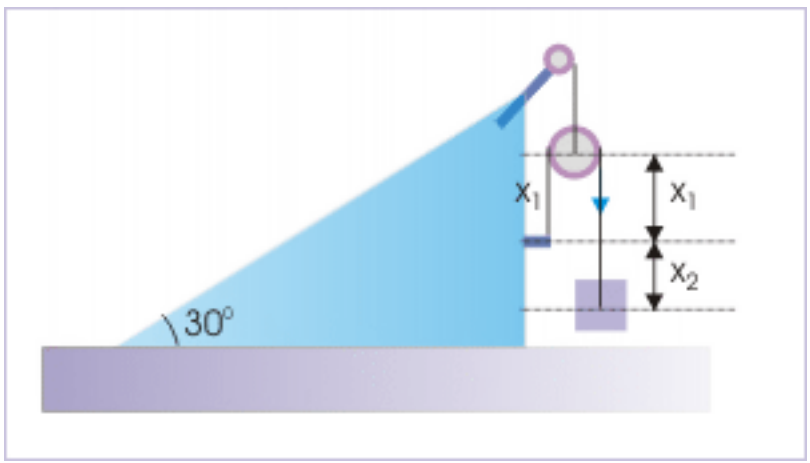

Figure 10: The constrain relation for positions are obtained by measuring positions with respect to a fixed point.

$$x_1 + x_1 + x_2 = L$$

Differentiating w.r.t time twice, we have :

$$\Rightarrow 2a_1 + a_2 = 0$$

Comparing magnitude only,

$$\Rightarrow a_2 = 2a_1$$

We see that if one end of the string is fixed, then the acceleration of the other end of the string passing over a movable "mass-less" pulley is two times the acceleration of the movable pulley. Let,

$$a_1 = a$$

$$a_2 = 2a$$

Substituting for tension and acceleration in the equations of the blocks as derived earlier, we have

$$2T_2 - mg\sin 30^0 = ma$$

$$nmg - T_2 = 2nma$$

Multiplying second equation with 2 and adding to eliminate " T_2 ",

$$2nmg - \frac{mg}{2} = a\left(m + 4nm\right)$$

$$\Rightarrow a = \frac{\left(4n - 1\right)g}{2\left(1 + 4n\right)}$$

The acceleration of the block of mass "nm" is :

$$\Rightarrow a_2 = 2a = \frac{\left(4n - 1\right)g}{\left(1 + 4n\right)}$$

3.2

Problem 4 : In the arrangement shown, the masses are $m_1 = 2$ kg and $m_2 = 4$ kg. The mass of the strings and pulleys are negligible. The friction is negligible between block and fixed incline and also between pulley and string. Find the accelerations of two blocks.

Incline and pulley system

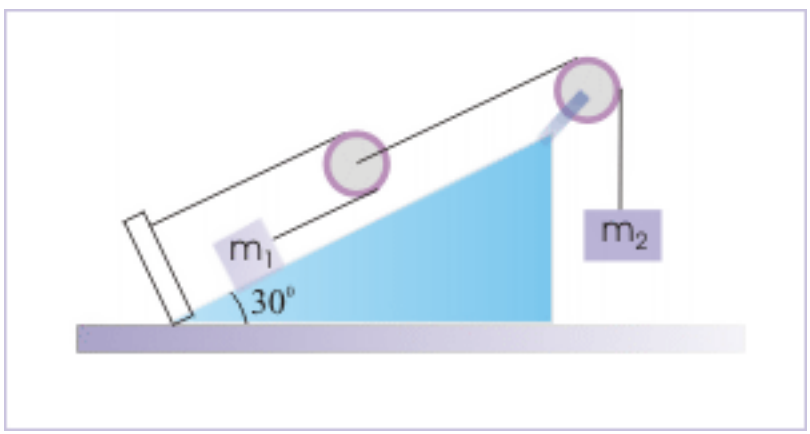

Figure 11: Two Blocks are attached to two moving pulley.

Solution : Considering free body diagram of the moving pulley, we see that tension in the string attached to mass " m_1" is half of the tension attached to the moving pulley. Hence, if "T" be the tension in the string attached to block of mass " m_1 ", then the tension in the string attached to the pulley is "2T".

Incline and pulley system

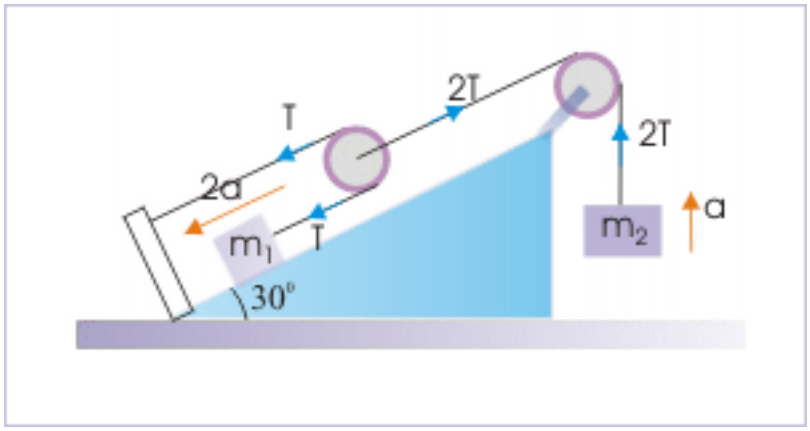

Figure 12: Tensions and accelerations of the elements of the system are shown.

From, the constrain relation as derived in prevous example, we know that the acceleration of the block of mass " m_1 " is twice that of the pulley. Hence, if "a" be the acceleration of the moving pulley, then acceleration of the block of mass " m_1 " is "2a".

The free body diagram of two blocks are shown here. The force analysis of the block of mass " m_1 " is :

Free body diagrams

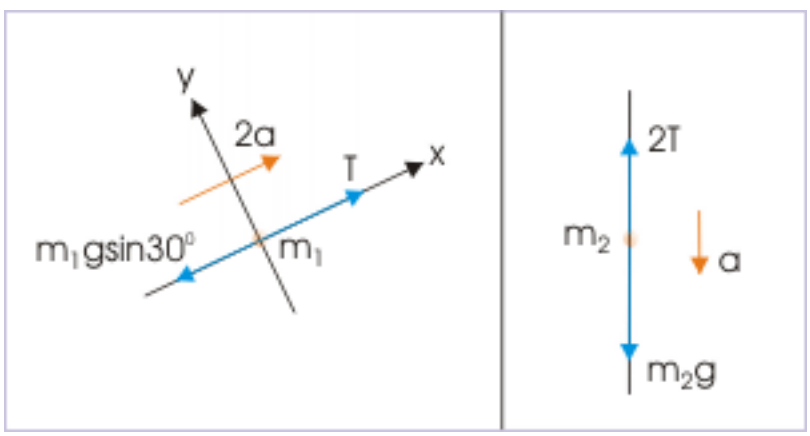

Figure 13: Free body diagrams of two blocks are shown.

$$T - m_1 g \sin 30^0 = m_1 a_1$$

$$\Rightarrow T - 2X10X\frac{1}{2} = 2X2a$$

$$T - 10 = 4a$$

Similarly, force analysis of the block of mass " m_2 " is :

$$m_2 g - 2T = m_2 a_2$$

$$\Rightarrow 40 - 2T = 4a$$

$$20 - T = 2a$$

Adding two equations, we have :

$$\Rightarrow 20 - 10 = 6a$$

$$\Rightarrow a = \frac{10}{6} = \frac{5}{3}$$

Thus, accelerations of block are :

$$a_2 = a = \frac{5}{3} \quad m/s^2$$

$$a_1 = 2a = \frac{10}{3} \quad m/s^2$$

4 Acknowledgment

Author wishes to thank John Trautwein for making suggestions to improve this module.