Gravitational field (application)*

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Abstract

Solving problems is an essential part of the understanding process.

Questions and their answers are presented here in the module text format as if it were an extension of the treatment of the topic. The idea is to provide a verbose explanation, detailing the application of theory. Solution presented is, therefore, treated as the part of the understanding process – not merely a Q/A session. The emphasis is to enforce ideas and concepts, which can not be completely absorbed unless they are put to real time situation.

1 Representative problems and their solutions

We discuss problems, which highlight certain aspects of the study leading to gravitational field. The questions are categorized in terms of the characterizing features of the subject matter :

- Gravitational field
- Gravitational force
- Superposition principle

2 Gravitational field

Problem 1 : Calculate gravitational field at a distance "r" from the center of a solid sphere of uniform density, "ρ", and radius "R". Given that r < R.

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Gravitational field

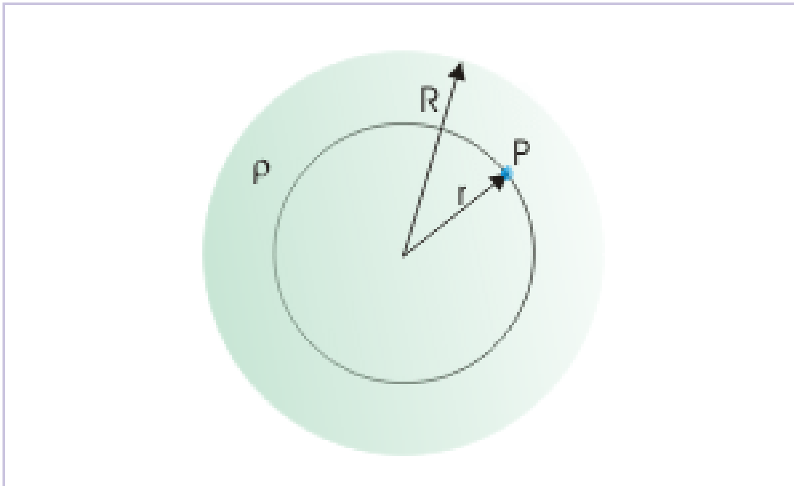

Figure 1: Gravitational field inside a solid sphere.

Solution : The point is inside the solid sphere of uniform density. We apply the theorem that gravitational field due to mass outside the sphere of radius "r" is zero at the point where field is being calculated. Let the mass of the sphere of radius "r" be "m", then :

$$m = \frac{4}{3}\pi r^3 \rho$$

The gravitational field due to this sphere on its surface is given by :

$$E = \frac{Gm}{r^2} = \frac{GX\frac{4}{3}\pi r^3 \rho}{r^2}$$

$$\Rightarrow E = \frac{4G\pi r\rho}{3}$$

3 Gravitational force

Problem 2 : A sphere of mass "2M" is placed a distance "$\sqrt{3}$ R" on the axis of a vertical ring of radius "R" and mass "M". Find the force of gravitation between two bodies.

Gravitational force

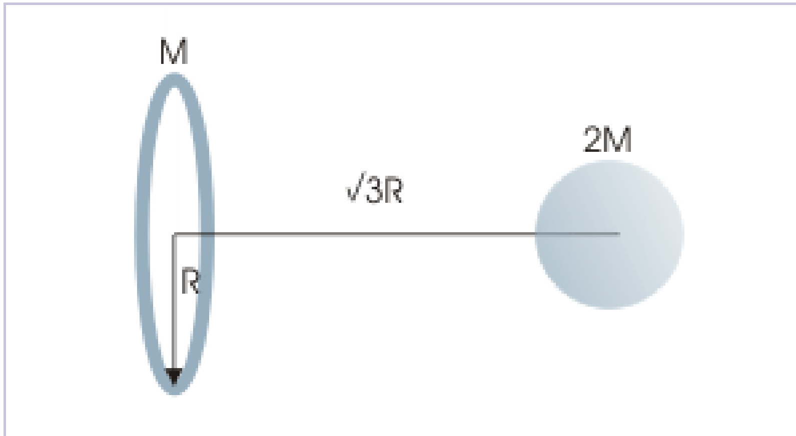

Figure 2: The center of sphere lies on the axis of ring.

Solution : Here, we determine gravitational field due to ring at the axial position, where center of sphere lies. Then, we multiply the gravitational field with the mass of the sphere to calculate gravitational force between two bodies.

The gravitational field due to ring on its axis is given as :

$$E = \frac{GMx}{\left(R^2 + x^2\right)^{\frac{3}{2}}}$$

Putting values,

$$\Rightarrow E = \frac{GM\sqrt{3}R}{\left\{R^2 + \left(\sqrt{3}R\right)^2\right\}^{\frac{3}{2}}}$$

$$\Rightarrow E = \frac{\sqrt{3}GM}{8R^2}$$

The sphere acts as a point mass. Therefore, the gravitational force between two bodies is :

$$\Rightarrow F = 2ME = \frac{2\sqrt{3}GM^2}{8R^2} = \frac{\sqrt{3}GM^2}{4R^2}$$

4 Superposition principle

4.1

Problem 3 : A spherical cavity is made in a solid sphere of mass "M" and radius "R" as shown in the figure. Find the gravitational field at the center of cavity due to remaining mass.

Superposition principle

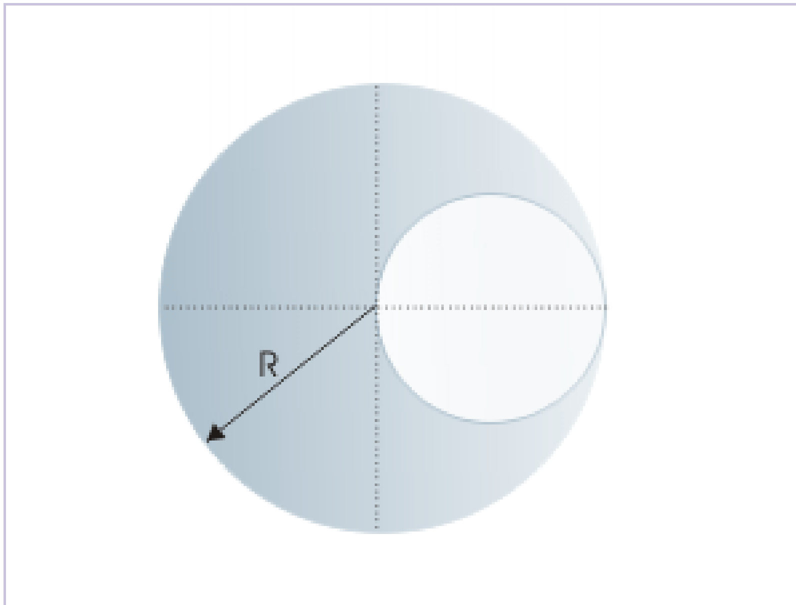

Figure 3: The gravitational field at the center of spherical cavity

Solution : According to superposition principle, gravitational field (**E**) due to whole mass is equal to vector sum of gravitational field due to remaining mass (E_1) and removed mass (E_2).

$$E = E_1 + E_2$$

The gravitation field due to a uniform solid sphere is zero at its center. Therefore, gravitational field due to removed mass is zero at its center. It means that gravitational field due to solid sphere is equal to gravitational field due to remaining mass. Now, we know that "**E**" at the point acts towards center of sphere. As such both "**E**" and " E_1 " acts along same direction. Hence, we can use scalar form,

$$E_1 = E$$

Now, gravitational field due to solid sphere of radius "R" at a point "r" within the sphere is given as :

$$E = \frac{GMr}{R^3}$$

Here,

Superposition principle

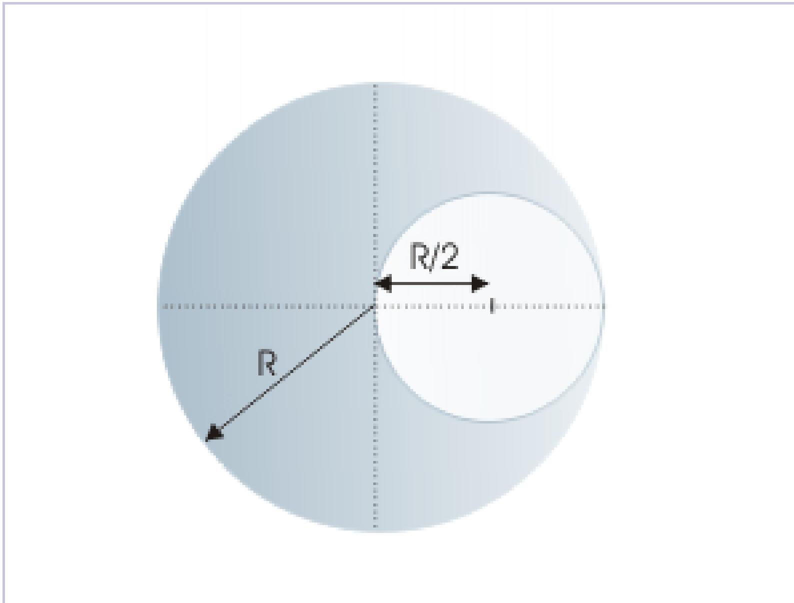

Figure 4: The gravitational field at the center of spherical cavity

$$r = R - \frac{R}{2} = \frac{R}{2}$$

Thus,

$$\Rightarrow E = \frac{GMR}{2R^3} = \frac{GM}{2R^2}$$

Therefore, gravitational field due to remaining mass, " E_1 ", is :

$$\Rightarrow E_1 = E = \frac{GM}{2R^2}$$

4.2

Problem 4 : Two concentric spherical shells of mass " m_1 " and " m_2 " have radii " r_1 " and " r_2 " respectively, where $r_2 > r_1$. Find gravitational intensity at a point, which is at a distance "r" from the common center for following situations, when it lies (i) inside smaller shell (ii) in between two shells and (iii) outside outer shell.

Solution : Three points "A", "B" and "C" corresponding to three given situations in the question are shown in the figure :

Superposition principle

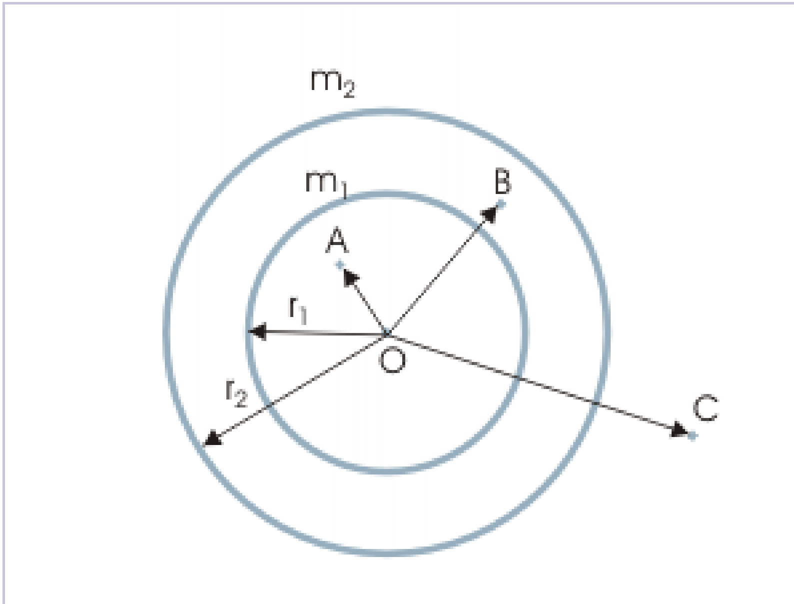

Figure 5: The gravitational field at three different points

The point inside smaller shell is also inside outer shell. The gravitational field inside a shell is zero. Hence, net gravitational field at a position inside the smaller shell is zero,

$$E_1 = 0$$

The gravitational field strength due to outer shell (E_o) at a point inside is zero. On the other hand, gravitational field strength due to inner shell (E_i) at a point outside is :

$$\Rightarrow E_i = \frac{GM}{r^2}$$

Hence, net gravitational field at position in between two shells is :

$$E_2 = E_i + E_o = \frac{Gm_1}{r^2}$$

A point outside outer shell is also outside inner shell. Hence, net field strength at a position outside outer shell is :

$$E_3 = Ei + Eo = \frac{Gm_1}{r^2} + \frac{Gm_2}{r^2}$$

$$\Rightarrow E_3 = \frac{G(m_1 + m_2)}{r^2}$$