

# FUNCTION OPERATIONS (EXERCISE)\*

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In this module, we shall work with different function types, which are combined in various ways to form a function. The domain of such functions are determined in accordance with rules for function operations.

## Working rules :

- Find domain of the each individual function, which composes the given function. The individual functions may be different function types.
- The domain of a function is unchanged when it is multiplied with a scalar (i.e. a constant)
- The resulting domain after addition, subtraction and multiplication of two functions is given by the intersection of domains, " $D = D_1 \cap D_2$ ".
- In the case of division, we need to remove values for which denominator is zero. Domain =  $D_1 \cap D_2 - \{\text{values of "x" for which denominator is zero}\}$ .

NOTE: This exercise module did not follow immediately after the module on function operations. We needed to know different function types first to apply the concept with them.

## 1

**Problem 1:** Find the domain of the function given by :

$$f(x) = \frac{x}{[x-2][x+1]}$$

### Solution :

**Statement of the problem :** The function has rational form. Denominator consists of product of two greatest integer functions.

We can consider, the function as product of three individual functions :

$$f(x) = x \times \frac{1}{[x-2]} \times \frac{1}{[x+1]}$$

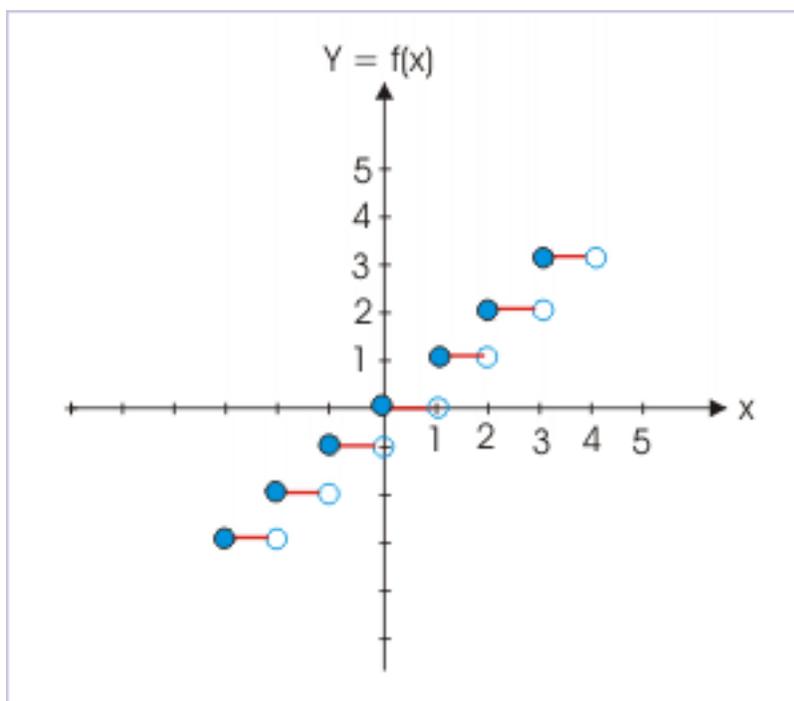
The domain of "x" is "R". We, now, analyze individual greatest integer functions such that it does not become zero. If we recall the graph of greatest integer function, then we can realize that the value of greatest integer [x] is equal to zero for the interval given by  $0 \leq x < 1$ . Following this clue, we find the intervals in which greatest integer functions are zero.

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### Greatest integer function



**Figure 1:** The greatest integer function evaluates to zero for  $0 \leq x < 1$ .

For  $[x - 2] = 0$ ,

$$[x - 2] = 0, \quad \text{if } 0 \leq x - 2 < 1$$

$$\Rightarrow [x - 2] = 0, \quad \text{if } 2 \leq x < 3$$

$$\Rightarrow [x - 2] = 0, \quad \text{if } x \in [2, 3)$$

It means that given function is undefined for this interval of "x". The domain of the function for this condition is :

$$D_1 = R - [2, 3)$$

Similarly, for  $[x + 1] = 0$

$$[x + 1] = 0, \quad \text{if } 0 \leq x + 1 < 1$$

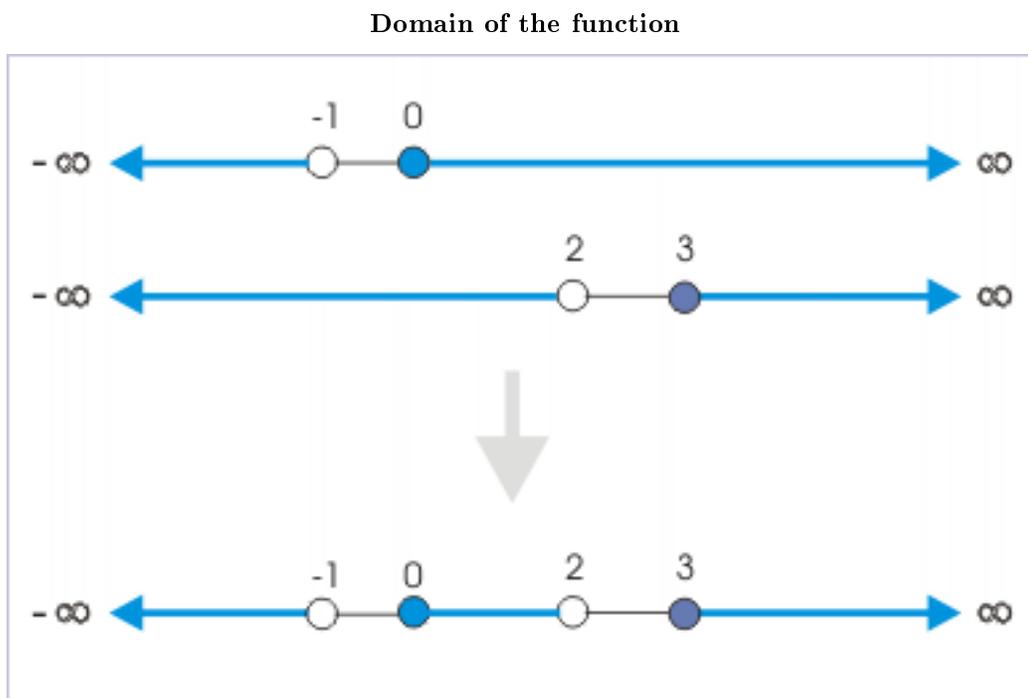
$$\Rightarrow [x + 1] = 0, \quad \text{if } -1 \leq x < 0$$

$$\Rightarrow [x + 1] = 0, \quad \text{if } x \in [-1, 0)$$

The domains for this condition is :

$$D_2 = \mathbb{R} - [-1, 0)$$

Hence, domain of the given function is intersection of two domains as shown in the figure. Note that we have not considered the domain of numerator, "x", as its domain is "R" and its intersection with any interval is interval itself.



**Figure 2:** The domain of the function is intersection of domains of individual functions.

$$\text{Domain} = D_1 \cap D_2$$

$$\text{Domain} = (-\infty, -1) \cup [0, 2) \cup [3, \infty)$$

**2**

**Problem 2:** Find the domain of the function given by :

$$f(x) = \log_e \cos(x - 5) + \sqrt{(9 - x^2)}$$

**Solution :**

**Statement of the problem :** The given function is sum of logarithmic and algebraic function.

Here, we observe that argument (input) of logarithmic function is itself a trigonometric function. We know that cosine function is real for all real values of "x". The important point to realize here is that we have to evaluate logarithmic function for the values of trigonometric function " $\cos(x-5)$ " – not for independent variable "x". Now, the argument of logarithmic function is a positive number. It means that :

$$\cos(x - 5) > 0$$

The basic interval of cosine function is  $[-\pi/2, \pi/2]$ . The solution of the cosine inequality is the domain of the logarithmic function :

$$D_1 = 2n\pi - \frac{\pi}{2} < (x - 5) < 2n\pi + \frac{\pi}{2}, \quad n \in Z$$

$$\Rightarrow D_1 = \left(2n - \frac{1}{2}\right)\pi + 5 < x < \left(2n + \frac{1}{2}\right)\pi + 5, \quad n \in Z$$

For the algebraic function, the expression within the square root is non-negative number :

$$\Rightarrow 9 - x^2 \geq 0 \quad \Rightarrow x^2 - 9 \leq 0 \quad \Rightarrow (x + 3)(x - 3) \leq 0$$

Clearly, roots of the quadratic equation, when equated to zero, is -3,3. Here, coefficient of quadratic equal equation is positive. Therefore, middle section is negative. Hence, its domain is :

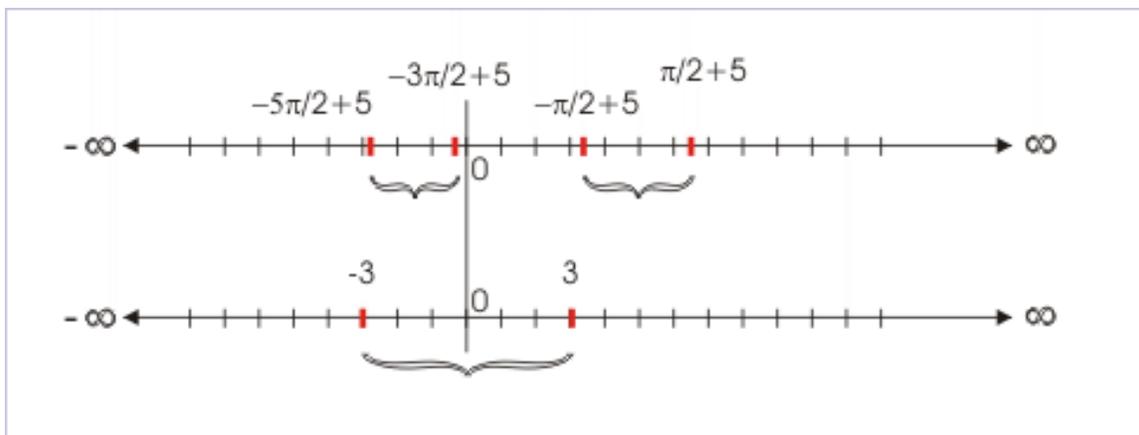
$$D_2 = -3 \leq x \leq 3$$

The domain of given function,  $f(x)$ , is intersection of two functions i.e.

$$\Rightarrow D = D_1 \cap D_2$$

From the figure, the common interval is between  $-5\pi/2$  and  $-3\pi/2$  as obtained for  $n = -1$ .

### Domain of the function



**Figure 3:** The domain of the function is intersection of domains of individual functions.

NOTE: We should draw a rough number line diagram on paper for few values of “n”.

$$D = \left(-\frac{5\pi}{2}, -\frac{3\pi}{2}\right)$$