Diversity Techniques*

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This section shows the error-performance improvements that can be obtained with the use of diversity techniques.

The bit-error-probability, $\overline{P_B}$, averaged through all the "ups and downs" of the fading experience in a slow-fading channel is as follows:

$\overline{P_B} = \int P_B(x)\, p(x)\, \mathrm{dx}$

where $P_B(x)$ is the bit-error probability for a given modulation scheme at a specific value of SNR $= x$, where $x = \alpha^2 E_b/N_0$, and $p(x)$ is the pdf of x due to the fading conditions. With E_b and N_0 constant, α is used to represent the amplitude variations due to fading.

For **Rayleigh fading**, α has a **Rayleigh distribution** so that α^2, and consequently x, have a **chi-squared distribution**:

$p(x) = \frac{1}{\Gamma}\exp\left(-\frac{x}{\Gamma}\right)\ x \geq 0$

where $\Gamma = \overline{\alpha^2} E_b/N_0$ is the SNR averaged through the "ups and downs" of fading. If each diversity (signal) branch, $i = 1, 2, ..., M$, has an instantaneous SNR $= \gamma_i$, and we assume that each branch has the same average SNR given by Γ, then

$p(\gamma_i) = \frac{1}{\Gamma}\exp\left(-\frac{\gamma_i}{\Gamma}\right)\ \gamma_i \geq 0$

The probability that a single branch has SNR less than some threshold γ is:

$P(\gamma_i \leq \gamma) = \int_0^\gamma p(\gamma_i)\,\mathrm{d}\gamma_i = \int_0^\gamma \frac{1}{\Gamma}\exp\left(-\frac{\gamma_i}{\Gamma}\right)\mathrm{d}\gamma_i$

$= 1 - \exp\left(-\frac{\gamma}{\Gamma}\right)$

The probability that all M independent signal diversity branches are received simultaneously with an SNR less than some threshold value γ is:

$P(\gamma_1, ..., \gamma_M \leq \gamma) = \left[1 - \exp\left(-\frac{\gamma}{\Gamma}\right)\right]^M$

The probability that any single branch achieves SNR $> \gamma$ is:

$P(\gamma_i > \gamma) = 1 - \left[1 - \exp\left(-\frac{\gamma}{\Gamma}\right)\right]^M$

This is the probability of exceeding a threshold when selection diversity is used.

Example: Benefits of Diversity

*Version 1.1: Nov 15, 2007 7:15 am US/Central

Assume that four-branch diversity is used, and that each branch receives an independently Rayleigh-fading signal. If the average SNR is $\Gamma = 20\,\mathrm{dB}$, determine the probability that all four branches are received simultaneously with an SNR less than $10\,\mathrm{dB}$ (and also, the probability that this threshold will be exceeded).

Compare the results to the case when no diversity is used.

Solution

With $\gamma = 10\,\mathrm{dB}$, and $\gamma/\Gamma = 10\,\mathrm{dB} - 20\,\mathrm{dB} = -10\,\mathrm{dB} = 0.1$, we solve for the probability that the SNR will drop below $10\,\mathrm{dB}$, as follows:

$P\left(\gamma_1, \gamma_2, \gamma_3, \gamma_4 \le 10\,\mathrm{dB}\right) = \left[1 - \exp\left(-0.1\right)\right]^4 = 8.2 \times 10^{-5}$

or, using selection diversity, we can say that

$P\left(\gamma_i > 10\,\mathrm{dB}\right) = 1 - 8.2 \times 10^{-5} = 0.9999$

Without diversity,

$P\left(\gamma_1 \le 10\,\mathrm{dB}\right) = \left[1 - \exp\left(-0.1\right)\right]^1 = 0.095$

$P\left(\gamma_1 > 10\,\mathrm{dB}\right) = 1 - 0.095 = 0.905$