DIVERSITY TECHNIQUES*

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This section shows the error-performance improvements that can be obtained with the use of diversity techniques.

The bit-error-probability, $\overline{P_B}$, averaged through all the "ups and downs" of the fading experience in a slow-fading channel is as follows:

 $\overline{P_B} = \int P_B(x) p(x) \,\mathrm{dx}$

where $P_B(x)$ is the bit-error probability for a given modulation scheme at a specific value of SNR = x, where $x = \alpha^2 E_b / N_0$, and p(x) is the pdf of x due to the fading conditions. With E_b and N_0 constant, α is used to represent the amplitude variations due to fading.

For **Rayleigh fading**, α has a **Rayleigh distribution** so that α^2 , and consequently x, have a chisquared distribution:

 $p(x) = \frac{1}{\Gamma} \exp\left(-\frac{x}{\Gamma}\right) x \ge 0$

where $\Gamma = \overline{\alpha^2} E_b / N_0$ is the SNR averaged through the "ups and downs" of fading. If each diversity (signal) branch, i = 1, 2, ..., M, has an instantaneous SNR $= \gamma_i$, and we assume that each branch has the same average SNR given by Γ , then

 $p(\gamma_i) = \frac{1}{\Gamma} \exp\left(-\frac{\gamma_i}{\Gamma}\right) \gamma_i \ge 0$

The probability that a single branch has SNR less than some threshold γ is: $P(\gamma_i \leq \gamma) = \int_0^{\gamma} p(\gamma_i) \, \mathrm{d}\gamma_i = \int_0^{\gamma} \frac{1}{\Gamma} \exp\left(-\frac{\gamma_i}{\Gamma}\right) \, \mathrm{d}\gamma_i$ $= 1 - \exp\left(-\frac{\gamma}{\Gamma}\right)$

The probability that all M independent signal diversity branches are received simultaneously with an SNR less than some threshold value γ is:

 $P(\gamma_1, ..., \gamma_M \leq \gamma) = \left[1 - \exp\left(-\frac{\gamma}{\Gamma}\right)\right]^M$ The probability that any single branch achieves SNR > γ is:

 $P(\gamma_i > \gamma) = 1 - \left[1 - \exp\left(-\frac{\gamma}{\Gamma}\right)\right]^M$

This is the probability of exceeding a threshold when selection diversity is used.

Example: Benefits of Diversity

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Assume that four-branch diversity is used, and that each branch receives an independently Rayleighfading signal. If the average SNR is $\Gamma = 20 \text{ dB}$, determine the probability that all four branches are received simultaneously with an SNR less than 10 dB (and also, the probability that this threshold will be exceeded).

Compare the results to the case when no diversity is used. Solution

With $\gamma = 10 \,\mathrm{dB}$, and $\gamma/\Gamma = 10 \,\mathrm{dB} - 20 \,\mathrm{dB} = -10 \,\mathrm{dB} = 0.1$, we solve for the probability that the SNR will drop below $10 \,\mathrm{dB}$, as follows: $P(\gamma_1, \gamma_2, \gamma_3, \gamma_4 \le 10 \,\mathrm{dB}) = [1 - \exp(-0.1)]^4 = 8.2 \times 10^{-5}$ or, using selection diversity, we can say that $P(\gamma_i > 10 \,\mathrm{dB}) = 1 - 8.2 \times 10^{-5} = 0.9999$ Without diversity, $P(\gamma_1 \le 10 \,\mathrm{dB}) = [1 - \exp(-0.1)]^1 = 0.095$ $P(\gamma_1 > 10 \,\mathrm{dB}) = 1 - 0.095 = 0.905$