Sonar Receiver Model*

Laurence Riddle

Abstract

This module describes the random process statistics of a typical sonar receiver, where the sonar data is frequency shifted and then sampled to a digital time series.

1 Active Sonar Receiver Model

Consider the sonar receiver processing chain shown below:

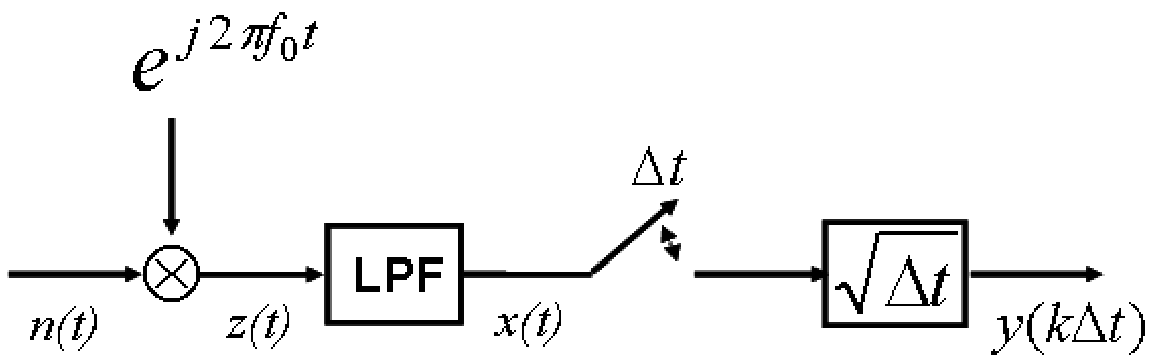

Figure 1

The sonar array input is heterodyned, low-pass filtered, sampled and scaled to generate a discrete time set of samples of the noise plus signal.

The input noise $n(t)$is a real-valued, wide sense stationary random process with power spectral density $P_{\mathrm{nn}}(f)$. Because $n(t)$is wide sense stationary, the power spectral density and the autocorrelation function are related by

$$R_{\mathrm{nn}}(\tau) = \int_{-\infty}^{\infty} P_{\mathrm{nn}}(f)\, e^{-\mathrm{j}2\pi\mathrm{f}\tau}\mathrm{df}$$

*Version 1.2: Dec 4, 2007 12:40 pm US/Central
†http://creativecommons.org/licenses/by/2.0/

We assume that $n(t)$is zero mean. A complex carrier $e^{j2\pi ft}$is applied to the random process $n(t)$resulting in a complex random process $z(t)$. The mean value of $z(t)$is given by:

$E\{z(t)\} = E\{n(t)e^{j2\pi ft}\} = E\{n(t)\}e^{j2\pi ft} = 0$

The covariance of $z(t)$is given by

$E\{z(t)z(s)\} = e^{j2\pi f(t-s)}E\{n(t)n(s)\} = e^{j2\pi f(t-s)}R_{nn}(t-s)$,

which shows that $z(t)$is wide sense stationary as well.

$z(t)$is passed through a band-pass filter to produce $x(t)$. The frequency response of the band-pass filter is assumed to be low-pass with a bandwidth of B/2 Hertz. That is we will assume that the filter transfer function $H_{BPF}(f)$is given by:

$$H(f) = \{ \begin{array}{c} 1, \mid f \mid < B/2 \\ 0, \mid f \mid > B/2 \end{array}$$

The resulting $x(t)$is a wide sense stationary random process with zero-mean and power spectral density:

$$P_{xx}(f) \approx \{ \begin{array}{c} N_0/2, \mid f \mid < B/2 \\ 0, \mid f \mid > B/2 \end{array}, (1)$$

Where we have assumed that the bandwidth of the receiver is small relative to the center frequency of the signal we are trying to detect, $B/f_0 \ll 1$. The power spectral density of $x(t)$can then be approximated by the power spectral density of the noise near f_0:

$P_{nn}(f) \approx P_{nn}(f_0) = N_0/2, \mid f - f_0 \mid < B/2$

If $x(t)$ has a power spectral density given by Eq-1, then the autocorrelation function of $x(t)$ becomes:

$R_{xx}(\tau) = E\{x(t)x(t+\tau)\} = \int_{-B/2}^{B/2} \frac{N_0}{2} e^{j2\pi f\tau} df = \frac{N_0}{2} \frac{\sin \pi B\tau}{\pi\tau}$

Note that $R_{xx}(0) = \frac{N_0 B}{2}$.

Now if we choose a sampling interval $\Delta t = 1/B$; then the samples at $k\Delta t$ have an autocorrelation given by

$E\{x(k\Delta t)x(l\Delta t)\} = \frac{N_0}{2} \frac{\sin(\pi B(k-l)\Delta t)}{\pi(k-l)\Delta t} = \frac{BN_0}{2}\delta_{kl}$,

Hence $x(k\Delta t), k = 0, 1, ...$ is a discrete time, wide sense stationary, white noise with intensity $\frac{BN_0}{2}$.

For matched filtering applications, we scale the output of the Analog to Digital conversion process by $\Delta t = 1/B$to conserve the signal energy over a time interval T. This creates the discrete time process $y_k = x(k\Delta t)\sqrt{\Delta t}$.

To see this, consider that

$E\{\int_0^T \mid x(t) \mid^2 dt\} = \int_0^T E\{\mid x(t) \mid^2\} dt = \int_0^T R_{xx}(0) dt = \frac{BTN_0}{2}$

And

$E\{\sum_{k=1}^{k=\frac{T}{\Delta t}} \mid y(k) \mid^2\} = \sum_{k=1}^{k=\frac{T}{\Delta t}} E\{\mid y(k) \mid^2\} = \sum_{k=1}^{k=\frac{T}{\Delta t}} \frac{BN_0}{2}\Delta t = \frac{BTN_0}{2}$.