

CONCLUSIONS: FAST FOURIER TRANSFORMS*

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This book has developed a class of efficient algorithms based on index mapping and polynomial algebra. This provides a framework from which the Cooley-Tukey FFT, the split-radix FFT, the PFA, and WFTA can be derived. Even the programs implementing these algorithms can have a similar structure. Winograd's theorems were presented and shown to be very powerful in both deriving algorithms and in evaluating them. The simple radix-2 FFT provides a compact, elegant means for efficiently calculating the DFT. If some elaboration is allowed, significant improvement can be had from the split-radix FFT, the radix-4 FFT or the PFA. If multiplications are expensive, the WFTA requires the least of all.

Several methods for transforming real data were described that are more efficient than directly using a complex FFT. A complex FFT can be used for real data by artificially creating a complex input from two sections of real input. An alternative and slightly more efficient method is to construct a special FFT that utilizes the symmetries at each stage.

As computers move to multiprocessors and multicore, writing and maintaining efficient programs becomes more and more difficult. The highly structured form of FFTs allows automatic generation of very efficient programs that are tailored specifically to a particular DSP or computer architecture.

For high-speed convolution, the traditional use of the FFT or PFA with blocking is probably the fastest method although rectangular transforms, distributed arithmetic, or number theoretic transforms may have a future with special VLSI hardware.

The ideas presented in these notes can also be applied to the calculation of the discrete Hartley transform [6], [2], the discrete cosine transform [3], [7], and to number theoretic transforms [1], [4], [5].

There are many areas for future research. The relationship of hardware to algorithms, the proper use of multiple processors, the proper design and use of array processors and vector processors are all open. There are still many unanswered questions in multi-dimensional algorithms where a simple extension of one-dimensional methods will not suffice.

References

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