CONTINUOUS RANDOM VARIABLES: CONTINUOUS PROBABILITY FUNCTIONS^{*}

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Abstract

This module introduces the continuous probability function and explores the relationship between the probability of X and the area under the curve of f(X).

We begin by defining a continuous probability density function. We use the function notation f(x). Intermediate algebra may have been your first formal introduction to functions. In the study of probability, the functions we study are special. We define the function f(x) so that the area between it and the x-axis is equal to a probability. Since the maximum probability is one, the maximum area is also one.

For continuous probability distributions, PROBABILITY = AREA.

Example 1

Consider the function $f(x) = \frac{1}{20}$ for $0 \le x \le 20$. x = a real number. The graph of $f(x) = \frac{1}{20}$ is a horizontal line. However, since $0 \le x \le 20$, f(x) is restricted to the portion between x = 0 and x = 20, inclusive.



 $f(x) = \frac{1}{20}$ for $0 \le x \le 20$. The graph of $f(x) = \frac{1}{20}$ is a horizontal line segment when $0 \le x \le 20$. The area between $f(x) = \frac{1}{20}$ where $0 \le x \le 20$ and the x-axis is the area of a rectangle with base = 20 and height $=\frac{1}{20}$.

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 $AREA = 20 \cdot \frac{1}{20} = 1$

This particular function, where we have restricted x so that the area between the function and the x-axis is 1, is an example of a continuous probability density function. It is used as a tool to calculate probabilities.

Suppose we want to find the area between $f(x) = \frac{1}{20}$ and the x-axis where 0 < x < 2



 $AREA = (2 - 0) \cdot \frac{1}{20} = 0.1$ (2 - 0) = 2 = base of a rectangle $\frac{1}{20} = the height.$

The area corresponds to a probability. The probability that x is between 0 and 2 is 0.1, which can be written mathematically as P(0 < x < 2) = P(x < 2) = 0.1.

Suppose we want to find the area between $f(x) = \frac{1}{20}$ and the x-axis where 4 < x < 15



 $AREA = (15 - 4) \cdot \frac{1}{20} = 0.55$ (15 - 4) = 11 = the base of a rectangle $\frac{1}{20} = the height.$

The area corresponds to the probability P(4 < x < 15) = 0.55.

Suppose we want to find P(x = 15). On an x-y graph, x = 15 is a vertical line. A vertical line has no width (or 0 width). Therefore, $P(x = 15) = (\text{base})(\text{height}) = (0)(\frac{1}{20}) = 0$.



 $P(X \le x)$ (can be written as P(X < x) for continuous distributions) is called the cumulative distribution function or CDF. Notice the "less than or equal to" symbol. We can use the CDF to calculate P(X > x). The CDF gives "area to the left" and P(X > x) gives "area to the right." We calculate P(X > x) for continuous distributions as follows: P(X > x) = 1 - P(X < x).



Label the graph with f (x) and x. Scale the x and y axes with the maximum x and y values. $f(x) = \frac{1}{20}, 0 \le x \le 20.$



 $P(2.3 < x < 12.7) = (base) (height) = (12.7 - 2.3) \left(\frac{1}{20}\right) = 0.52$