Continuous Random Variables: Continuous Probability Functions[*]

Susan Dean
Barbara Illowsky, Ph.D.

Susan Dean

Barbara Illowsky, Ph.D.

This work is produced by The Connexions Project and licensed under the
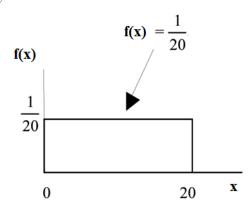Creative Commons Attribution License [†]

Abstract

This module introduces the continuous probability function and explores the relationship between the probability of X and the area under the curve of f(X).

We begin by defining a continuous probability density function. We use the function notation $f(x)$. Intermediate algebra may have been your first formal introduction to functions. In the study of probability, the functions we study are special. We define the function $f(x)$ so that the area between it and the x-axis is equal to a probability. Since the maximum probability is one, the maximum area is also one.

For continuous probability distributions, PROBABILITY = AREA.

Example 1

Consider the function $f(x) = \frac{1}{20}$ for $0 \leq x \leq 20$. x = a real number. The graph of $f(x) = \frac{1}{20}$ is a horizontal line. However, since $0 \leq x \leq 20$, $f(x)$ is restricted to the portion between $x = 0$ and $x = 20$, inclusive .

$f(x) = \frac{1}{20}$ **for** $0 \leq x \leq 20$.

The graph of $f(x) = \frac{1}{20}$ is a horizontal line segment when $0 \leq x \leq 20$.

The area between $f(x) = \frac{1}{20}$ where $0 \leq x \leq 20$ and the x-axis is the area of a rectangle with base = 20 and height $= \frac{1}{20}$.
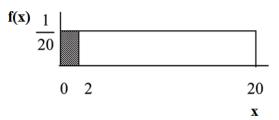
---

[*]Version 1.9: May 25, 2012 6:20 am -0500

[†]http://creativecommons.org/licenses/by/3.0/

AREA $= 20 \cdot \frac{1}{20} = 1$

This particular function, where we have restricted x so that the area between the function and the x-axis is 1, is an example of a continuous probability density function. It is used as a tool to calculate probabilities.

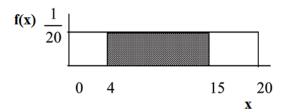**Suppose we want to find the area between** $f(x) = \frac{1}{20}$ **and the x-axis where** $0 < x < 2$

.

AREA $= (2 - 0) \cdot \frac{1}{20} = 0.1$

$(2 - 0) = 2 =$ base of a rectangle

$\frac{1}{20} =$ the height.

The area corresponds to a probability. The probability that x is between 0 and 2 is 0.1, which can be written mathematically as P(0<x<2) = P(x<2) = 0.1.

Suppose we want to find the area between $f(x) = \frac{1}{20}$ **and the x-axis where** $4 < x < 15$

.

AREA $= (15 - 4) \cdot \frac{1}{20} = 0.55$

$(15 - 4) = 11 =$ the base of a rectangle
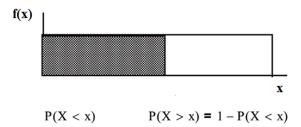
$\frac{1}{20} =$ the height.

The area corresponds to the probability $P(4 < x < 15) = 0.55$.

Suppose we want to find $P(x = 15)$**.** On an x-y graph, $x = 15$ is a vertical line. A vertical line has no width (or 0 width). Therefore, $P(x = 15) = (\text{base})(\text{height}) = (0)\left(\frac{1}{20}\right) = 0$.

$P(X \leq x)$ (can be written as $P(X < x)$ for continuous distributions) is called the cumulative distribution function or CDF. Notice the "less than or equal to" symbol. We can use the CDF to calculate $P(X > x)$. The CDF gives "area to the left" and $P(X > x)$ gives "area to the right." We calculate $P(X > x)$ for continuous distributions as follows: $P(X > x) = 1 - P(X < x)$.

f(x)

P(X < x)   P(X > x) = 1 − P(X < x)

Label the graph with f (x) and x. Scale the x and y axes with the maximum x and y values. $f(x) = \frac{1}{20}$, $0 \leq x \leq 20$.

f(x)

0   2.3   12.7   x

$P(2.3 < x < 12.7) = (\text{base})(\text{height}) = (12.7 - 2.3)\left(\frac{1}{20}\right) = 0.52$