Discrete Random Variables: Poisson (optional)*

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Abstract

This module describes the characteristics of a Poisson experiment and the Poisson probability distribution. This module is included in the Elementary Statistics textbook/collection as an optional lesson.

Characteristics of a Poisson experiment:

1. The Poisson gives the probability of a number of events occurring in a **fixed interval** of time or space if these events happen with a known average rate and independently of the time since the last event. For example, a book editor might be interested in the number of words spelled incorrectly in a particular book. It might be that, on the average, there are 5 words spelled incorrectly in 100 pages. The interval is the 100 pages.
2. The Poisson may be used to approximate the binomial if the probability of success is "small" (such as 0.01) and the number of trials is "large" (such as 1000). You will verify the relationship in the homework exercises. n is the number of trials and p is the probability of a "success."

Poisson probability distribution. The random variable X = the number of occurrences in the interval of interest. The mean and variance are given in the summary.

Example 1

The average number of loaves of bread put on a shelf in a bakery in a half-hour period is 12. Of interest is the number of loaves of bread put on the shelf in 5 minutes. The time interval of interest is 5 minutes. What is the probability that the number of loaves, selected randomly, put on the shelf in 5 minutes is 3?

Let X = the number of loaves of bread put on the shelf in 5 minutes. If the average number of loaves put on the shelf in 30 minutes (half-hour) is 12, **then the average number of loaves put on the shelf in 5 minutes is**

$\left(\frac{5}{30}\right) \cdot 12 = 2$ loaves of bread

The probability question asks you to find P (x = 3).

Example 2

A certain bank expects to receive 6 bad checks per day, on average. What is the probability of the bank getting fewer than 5 bad checks on any given day? Of interest is the number of checks the

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bank receives in 1 day, so the time interval of interest is 1 day. Let X = the number of bad checks the bank receives in one day. If the bank expects to receive 6 bad checks per day then the average is 6 checks per day. The probability question asks for $P(x < 5)$.

Example 3

You notice that a news reporter says "uh", on average, 2 times per broadcast. What is the probability that the news reporter says "uh" more than 2 times per broadcast.

This is a Poisson problem because you are interested in knowing the number of times the news reporter says "uh" during a broadcast.

Problem 1 ***(Solution on p. 4.)***

What is the interval of interest?

Problem 2 ***(Solution on p. 4.)***

What is the average number of times the news reporter says "uh" during one broadcast?

Problem 3 ***(Solution on p. 4.)***

Let X = _____. What values does X take on?

Problem 4 ***(Solution on p. 4.)***

The probability question is P(_____).

1 Notation for the Poisson: P = Poisson Probability Distribution Function

$X \sim \mathrm{P}(\mu)$

Read this as "X is a random variable with a Poisson distribution." The parameter is μ (or λ). μ (or λ) = the mean for the interval of interest.

Example 4

Leah's answering machine receives about 6 telephone calls between 8 a.m. and 10 a.m. What is the probability that Leah receives more than 1 call **in the next 15 minutes?**

Let X = the number of calls Leah receives in 15 minutes. (The **interval of interest** is 15 minutes or $\frac{1}{4}$ hour.)

$x = 0, 1, 2, 3, ...$

If Leah receives, on the average, 6 telephone calls in 2 hours, and there are eight 15 minutes intervals in 2 hours, then Leah receives

$\frac{1}{8} \cdot 6 = 0.75$

calls in 15 minutes, on the average. So, $\mu = 0.75$ for this problem.

$X \sim \mathrm{P}(0.75)$

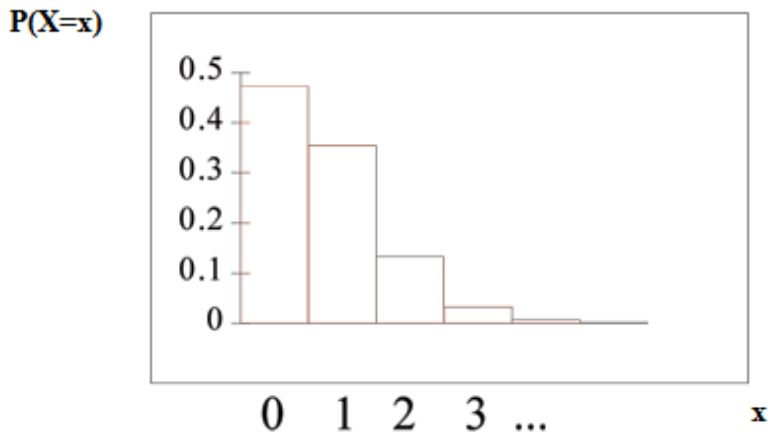
Find $P(x > 1)$. $P(x > 1) = 0.1734$ (calculator or computer)

TI-83+ and TI-84: For a general discussion, see **this example (Binomial)**. The syntax is similar. The Poisson parameter list is (μ for the interval of interest, number). **For this problem:**

Press 1- and then press 2nd DISTR. Arrow down to C:poissoncdf. Press ENTER. Enter .75,1). The result is $P(x > 1) = 0.1734$. NOTE: The TI calculators use λ (lambda) for the mean.

The probability that Leah receives more than 1 telephone call in the next fifteen minutes is about 0.1734.

The graph of $X \sim \mathrm{P}(0.75)$ is:



The y-axis contains the probability of x where X = the number of calls in 15 minutes.

Solutions to Exercises in this Module

Solution to Example 3, Problem 1 (p. 2)
One broadcast
Solution to Example 3, Problem 2 (p. 2)
2
Solution to Example 3, Problem 3 (p. 2)
Let $X =$ **the number of times the news reporter says "uh" during one broadcast**.
$x = 0, 1, 2, 3, ...$
Solution to Example 3, Problem 4 (p. 2)
P(x > 2)

Glossary

Definition 1: Poisson Distribution
A discrete random variable (RV) that counts the number of times a certain event will occur in a specific interval. Characteristics of the variable:

- The probability that the event occurs in a given interval is the same for all intervals.
- The events occur with a known mean and independently of the time since the last event.

The distribution is defined by the mean μ of the event in the interval. Notation: $X \tilde{} P(\mu)$. The mean is $\mu =$ np. The standard deviation is $\sigma = \sqrt{\mu}$. The probability of having exactly x successes in r trials is $P(X = x) = e^{-\mu} \frac{\mu^x}{x!}$. The Poisson distribution is often used to approximate the binomial distribution when n is "large" and p is "small" (a general rule is that n should be greater than or equal to 20 and p should be less than or equal to .05).