Probability Topics: Tree Diagrams (optional)*

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Abstract

This module introduces tree diagrams as a method for making some probability problems easier to solve. This module is included in the Elementary Statistics textbook/collection as an optional lesson.

A **tree diagram** is a special type of graph used to determine the outcomes of an experiment. It consists of "branches" that are labeled with either frequencies or probabilities. Tree diagrams can make some probability problems easier to visualize and solve. The following example illustrates how to use a tree diagram.

Example 1

In an urn, there are 11 balls. Three balls are red (R) and 8 balls are blue (B). Draw two balls, one at a time, **with replacement**. "With replacement" means that you put the first ball back in the urn before you select the second ball. The tree diagram using frequencies that show all the possible outcomes follows.

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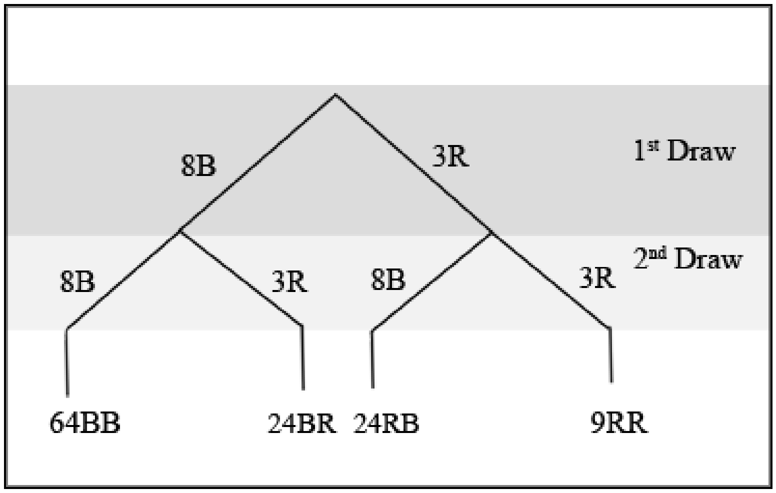


Figure 1: Total $= 64 + 24 + 24 + 9 = 121$

The first set of branches represents the first draw. The second set of branches represents the second draw. Each of the outcomes is distinct. In fact, we can list each red ball as R1, R2, and R3 and each blue ball as B1, B2, B3, B4, B5, B6, B7, and B8. Then the 9 RR outcomes can be written as:

R1R1 ; R1R2 ; R1R3 ; R2R1 ; R2R2 ; R2R3 ; R3R1 ; R3R2 ; R3R3

The other outcomes are similar.

There are a total of 11 balls in the urn. Draw two balls, one at a time, and with replacement. There are $11 \cdot 11 = 121$ outcomes, the size of the **sample space**.

Problem 1 ***(Solution on p. 5.)***

List the 24 BR outcomes: B1R1, B1R2, B1R3, ...

Problem 2

Using the tree diagram, calculate P(RR).

Solution

$\text{P(RR)} = \frac{3}{11} \cdot \frac{3}{11} = \frac{9}{121}$

Problem 3

Using the tree diagram, calculate P(RB OR BR).

Solution

$\text{P(RB OR BR)} = \frac{3}{11} \cdot \frac{8}{11} + \frac{8}{11} \cdot \frac{3}{11} = \frac{48}{121}$

Problem 4

Using the tree diagram, calculate P(R on 1st draw AND B on 2nd draw).

Solution

$\text{P(R on 1st draw AND B on 2nd draw)} = \text{P(RB)} = \frac{3}{11} \cdot \frac{8}{11} = \frac{24}{121}$

Problem 5

Using the tree diagram, calculate P(R on 2nd draw given B on 1st draw).

Solution

P(R on 2nd draw given B on 1st draw) $= \text{P(R on 2nd} \mid \text{B on 1st)} = \frac{24}{88} = \frac{3}{11}$

This problem is a conditional. The sample space has been reduced to those outcomes that already have a blue on the first draw. There are $24 + 64 = 88$ possible outcomes (24 BR and 64 BB). Twenty-four of the 88 possible outcomes are BR. $\frac{24}{88} = \frac{3}{11}$.

Problem 6 *(Solution on p. 5.)*

Using the tree diagram, calculate P(BB).

Problem 7 *(Solution on p. 5.)*

Using the tree diagram, calculate P(B on the 2nd draw given R on the first draw).

Example 2

An urn has 3 red marbles and 8 blue marbles in it. Draw two marbles, one at a time, this time without replacement from the urn. **"Without replacement"** means that you do not put the first ball back before you select the second ball. Below is a tree diagram. The branches are labeled with probabilities instead of frequencies. The numbers at the ends of the branches are calculated by multiplying the numbers on the two corresponding branches, for example, $\frac{3}{11} \cdot \frac{2}{10} = \frac{6}{110}$.

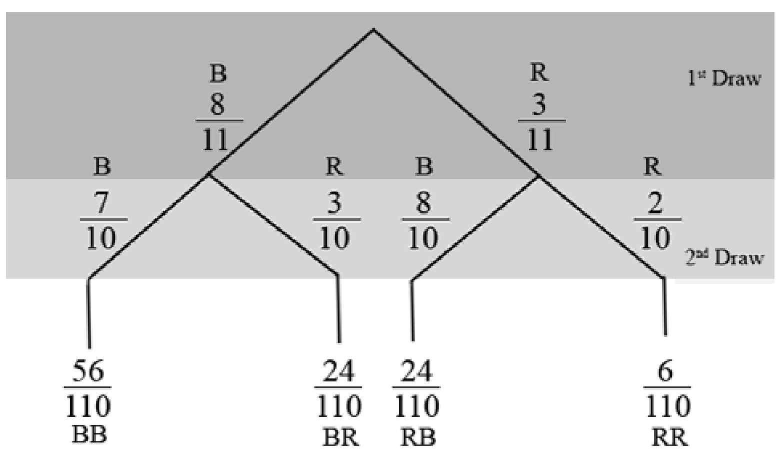


Figure 2: Total $= \frac{56 + 24 + 24 + 6}{110} = \frac{110}{110} = 1$

NOTE: If you draw a red on the first draw from the 3 red possibilities, there are 2 red left to draw on the second draw. You do not put back or replace the first ball after you have drawn it. You draw **without replacement**, so that on the second draw there are 10 marbles left in the urn.

Calculate the following probabilities using the tree diagram.

Problem 1

P(RR) =

Solution

P(RR) = $\frac{3}{11} \cdot \frac{2}{10} = \frac{6}{110}$

Problem 2 *(Solution on p. 5.)*

Fill in the blanks:

P(RB OR BR) = $\frac{3}{11} \cdot \frac{8}{10} + (\_\_\_)(\_\_\_) = \frac{48}{110}$

Problem 3 *(Solution on p. 5.)*

P(R on 2d | B on 1st) =

Problem 4 *(Solution on p. 5.)*

Fill in the blanks:

P(R on 1st and B on 2nd) = P(RB) = $(\_\_\_)(\_\_\_) = \frac{24}{110}$

Problem 5 *(Solution on p. 5.)*

P(BB) =

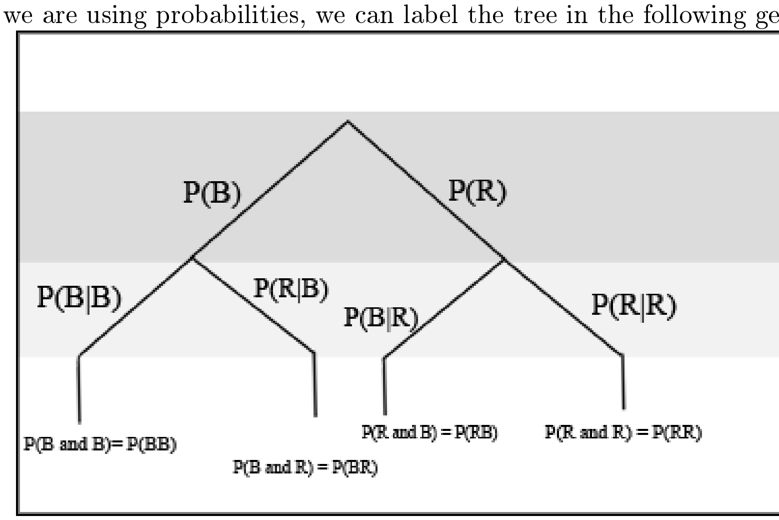
Problem 6

P(B on 2nd | R on 1st) =

Solution

There are 6 + 24 outcomes that have R on the first draw (6 RR and 24 RB). The 6 and the 24 are frequencies. They are also the numerators of the fractions $\frac{6}{110}$ and $\frac{24}{110}$. The sample space is no longer 110 but 6 + 24 = 30. Twenty-four of the 30 outcomes have B on the second draw. The probability is then $\frac{24}{30}$. Did you get this answer?

If we are using probabilities, we can label the tree in the following general way.



- P(R|R) here means P(R on 2nd | R on 1st)
- P(B|R) here means P(B on 2nd | R on 1st)
- P(R|B) here means P(R on 2nd | B on 1st)
- P(B|B) here means P(B on 2nd | B on 1st)

Solutions to Exercises in this Module

Solution to Example 1, Problem 1 (p. 2)
B1R1 ; B1R2 ; B1R3 ; B2R1 ; B2R2 ; B2R3 ; B3R1 ; B3R2 ; B3R3 ; B4R1 ; B4R2 ; B4R3 ; B5R1 ; B5R2 ; B5R3 ; B6R1 ; B6R2 ; B6R3 ; B7R1 ; B7R2 ; B7R3 ; B8R1 ; B8R2 ; B8R3

Solution to Example 1, Problem 6 (p. 3)
$\text{P(BB)} = \frac{64}{121}$

Solution to Example 1, Problem 7 (p. 3)
$\text{P(B on 2nd draw} \mid \text{R on 1st draw)} = \frac{8}{11}$

There are $9 + 24$ outcomes that have R on the first draw (9 RR and 24 RB). The sample space is then $9 + 24 = 33$. Twenty-four of the 33 outcomes have B on the second draw. The probability is then $\frac{24}{33}$.

Solution to Example 2, Problem 2 (p. 4)
$\text{P(RB or BR)} = \frac{3}{11} \cdot \frac{8}{10} + \left(\frac{8}{11}\right)\left(\frac{3}{10}\right) = \frac{48}{110}$

Solution to Example 2, Problem 3 (p. 4)
$\text{P(R on 2d} \mid \text{B on 1st)} = \frac{3}{10}$

Solution to Example 2, Problem 4 (p. 4)
$\text{P(R on 1st and B on 2nd)} = \text{P(RB)} = \left(\frac{3}{11}\right)\left(\frac{8}{10}\right) = \frac{24}{110}$

Solution to Example 2, Problem 5 (p. 4)
$\text{P(BB)} = \frac{8}{11} \cdot \frac{7}{10}$

Glossary

Definition 1: Sample Space
The set of all possible outcomes of an experiment.

Definition 2: Tree Diagram
The useful visual representation of a sample space and events in the form of a "tree" with branches marked by possible outcomes simultaneously with associated probabilities (frequencies, relative frequencies).