Probability Topics: Two Basic Rules of Probability*

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Abstract

This module introduces the multiplication and addition rules used when calculating probabilities.

1 The Multiplication Rule

If A and B are two events defined on a **sample space**, then: $\mathrm{P(A\ AND\ B)} = \mathrm{P(B)} \cdot \mathrm{P(A|B)}$.

This rule may also be written as : $\mathrm{P\,(A|B)} = \frac{P(A\ AND\ B)}{P(B)}$

(The probability of A given B equals the probability of A and B divided by the probability of B.)

If A and B are **independent**, then $\mathrm{P(A|B)} = \mathrm{P(A)}$. Then $\mathrm{P(A\ AND\ B)} = \mathrm{P(A|B)\ P(B)}$ becomes $\mathrm{P(A\ AND\ B)} = \mathrm{P(A)\ P(B)}$.

2 The Addition Rule

If A and B are defined on a sample space, then: $\mathrm{P(A\ OR\ B)} = \mathrm{P(A)} + \mathrm{P(B)} - \mathrm{P(A\ AND\ B)}$.

If A and B are **mutually exclusive**, then $\mathrm{P(A\ AND\ B)} = 0$. Then $\mathrm{P(A\ OR\ B)} = \mathrm{P(A)} + \mathrm{P(B)} - \mathrm{P(A\ AND\ B)}$ becomes $\mathrm{P(A\ OR\ B)} = \mathrm{P(A)} + \mathrm{P(B)}$.

Example 1

Klaus is trying to choose where to go on vacation. His two choices are: A = New Zealand and B = Alaska

- Klaus can only afford one vacation. The probability that he chooses A is $\mathrm{P(A)} = 0.6$ and the probability that he chooses B is $\mathrm{P(B)} = 0.35$.
- $\mathrm{P(A\ and\ B)} = 0$ because Klaus can only afford to take one vacation
- Therefore, the probability that he chooses either New Zealand or Alaska is $\mathrm{P(A\ OR\ B)} = \mathrm{P(A)} + \mathrm{P(B)} = 0.6 + 0.35 = 0.95$. Note that the probability that he does not choose to go anywhere on vacation must be 0.05.

Example 2

Carlos plays college soccer. He makes a goal 65% of the time he shoots. Carlos is going to attempt two goals in a row in the next game.

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A = the event Carlos is successful on his first attempt. P(A) = 0.65. B = the event Carlos is successful on his second attempt. P(B) = 0.65. Carlos tends to shoot in streaks. The probability that he makes the second goal **GIVEN** that he made the first goal is 0.90.

Problem 1
What is the probability that he makes both goals?

Solution
The problem is asking you to find P(A AND B) = P(B AND A). Since P(B|A) = 0.90:

$$\text{P(B AND A)} = \text{P(B|A) P(A)} = 0.90 * 0.65 = 0.585 \quad (1)$$

Carlos makes the first and second goals with probability 0.585.

Problem 2
What is the probability that Carlos makes either the first goal or the second goal?

Solution
The problem is asking you to find P(A OR B).

$$\text{P(A OR B)} = \text{P(A)} + \text{P(B)} - \text{P(A AND B)} = 0.65 + 0.65 - 0.585 = 0.715 \quad (2)$$

Carlos makes either the first goal or the second goal with probability 0.715.

Problem 3
Are A and B independent?

Solution
No, they are not, because P(B AND A) = 0.585.

$$\text{P(B)} \cdot \text{P(A)} = (0.65) \cdot (0.65) = 0.423 \quad (3)$$

$$0.423 \neq 0.585 = \text{P(B AND A)} \quad (4)$$

So, P(B AND A) is **not** equal to P(B) · P(A).

Problem 4
Are A and B mutually exclusive?

Solution
No, they are not because P(A and B) = 0.585.
To be mutually exclusive, P(A AND B) must equal 0.

Example 3
A community swim team has **150** members. **Seventy-five** of the members are advanced swimmers. **Forty-seven** of the members are intermediate swimmers. The remainder are novice swimmers. **Forty** of the advanced swimmers practice 4 times a week. **Thirty** of the intermediate swimmers practice 4 times a week. **Ten** of the novice swimmers practice 4 times a week. Suppose one member of the swim team is randomly chosen. Answer the questions (Verify the answers):

Problem 1

What is the probability that the member is a novice swimmer?

Solution

$\frac{28}{150}$

Problem 2

What is the probability that the member practices 4 times a week?

Solution

$\frac{80}{150}$

Problem 3

What is the probability that the member is an advanced swimmer and practices 4 times a week?

Solution

$\frac{40}{150}$

Problem 4

What is the probability that a member is an advanced swimmer and an intermediate swimmer? Are being an advanced swimmer and an intermediate swimmer mutually exclusive? Why or why not?

Solution

P(advanced AND intermediate) = 0, so these are mutually exclusive events. A swimmer cannot be an advanced swimmer and an intermediate swimmer at the same time.

Problem 5

Are being a novice swimmer and practicing 4 times a week independent events? Why or why not?

Solution

No, these are not independent events.

$$\text{P(novice AND practices 4 times per week)} = 0.0667 \tag{5}$$

$$\text{P(novice)} \cdot \text{P(practices 4 times per week)} = 0.0996 \tag{6}$$

$$0.0667 \neq 0.0996 \tag{7}$$

Example 4

Studies show that, if she lives to be 90, about 1 woman in 7 (approximately 14.3%) will develop breast cancer. Suppose that of those women who develop breast cancer, a test is negative 2% of the time. Also suppose that in the general population of women, the test for breast cancer is negative about 85% of the time. Let B = woman develops breast cancer and let N = tests negative. Suppose one woman is selected at random.

Problem 1
What is the probability that the woman develops breast cancer? What is the probability that woman tests negative?

Solution
P(B) = 0.143 ; P(N) = 0.85

Problem 2
Given that the woman has breast cancer, what is the probability that she tests negative?

Solution
P(N|B) = 0.02

Problem 3
What is the probability that the woman has breast cancer AND tests negative?

Solution
P(B AND N) = P(B) · P(N|B) = (0.143) · (0.02) = 0.0029

Problem 4
What is the probability that the woman has breast cancer or tests negative?

Solution
P(B OR N) = P(B) + P(N) − P(B AND N) = 0.143 + 0.85 − 0.0029 = 0.9901

Problem 5
Are having breast cancer and testing negative independent events?

Solution
No. P(N) = 0.85; P(N|B) = 0.02. So, P(N|B) does not equal P(N)

Problem 6
Are having breast cancer and testing negative mutually exclusive?

Solution
No. P(B AND N) = 0.0029. For B and N to be mutually exclusive, P(B AND N) must be 0.

Glossary

Definition 1: Independent Events
The occurrence of one event has no effect on the probability of the occurrence of any other event. Events A and B are independent if one of the following is true: (1). $P(A|B) = P(A)$; (2) $P(B|A) = P(B)$; (3) $P(A \text{and} B) = P(A)P(B)$.

Definition 2: Mutually Exclusive
An observation cannot fall into more than one class (category). Being in more than one category prevents being in a mutually exclusive category.

Definition 3: Sample Space
The set of all possible outcomes of an experiment.