

# NORMAL DISTRIBUTION: CALCULATIONS OF PROBABILITIES\*

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Probabilities are calculated by using technology. There are instructions in the chapter for the TI-83+ and TI-84 calculators.

NOTE: In the Table of Contents for **Collaborative Statistics**, entry **15. Tables** has a link to a table of normal probabilities. Use the probability tables if so desired, instead of a calculator. The tables include instructions for how to use them.

## Example 1

If the area to the left is 0.0228, then the area to the right is  $1 - 0.0228 = 0.9772$ .

## Example 2

The final exam scores in a statistics class were normally distributed with a mean of 63 and a standard deviation of 5.

## Problem 1

Find the probability that a randomly selected student scored more than 65 on the exam.

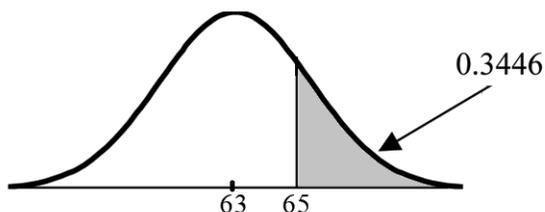
## Solution

Let  $X$  = a score on the final exam.  $X \sim N(63, 5)$ , where  $\mu = 63$  and  $\sigma = 5$

Draw a graph.

Then, find  $P(x > 65)$ .

$P(x > 65) = 0.3446$  (calculator or computer)



The probability that one student scores more than 65 is 0.3446.

Using the TI-83+ or the TI-84 calculators, the calculation is as follows. Go into 2nd DISTR.

After pressing 2nd DISTR, press 2:normalcdf.

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The syntax for the instructions are shown below.

normalcdf(lower value, upper value, mean, standard deviation) For this problem: normalcdf(65,1E99,63,5) = 0.3446. You get 1E99 (=  $10^{99}$ ) by pressing 1, the EE key (a 2nd key) and then 99. Or, you can enter  $10^{99}$  instead. The number  $10^{99}$  is way out in the right tail of the normal curve. We are calculating the area between 65 and  $10^{99}$ . In some instances, the lower number of the area might be -1E99 (=  $-10^{99}$ ). The number  $-10^{99}$  is way out in the left tail of the normal curve.

**HISTORICAL NOTE:** The TI probability program calculates a z-score and then the probability from the z-score. Before technology, the z-score was looked up in a standard normal probability table (because the math involved is too cumbersome) to find the probability. In this example, a standard normal table with area to the left of the z-score was used. You calculate the z-score and look up the area to the left. The probability is the area to the right.

$$z = \frac{65-63}{5} = 0.4 \quad . \text{ Area to the left is } 0.6554. \quad P(x > 65) = P(z > 0.4) = 1 - 0.6554 = 0.3446$$

### Problem 2

Find the probability that a randomly selected student scored less than 85.

#### Solution

Draw a graph.

Then find  $P(x < 85)$ . Shade the graph.  $P(x < 85) = 1$  (calculator or computer)

The probability that one student scores less than 85 is approximately 1 (or 100%).

The TI-instructions and answer are as follows:

$$\text{normalcdf}(0,85,63,5) = 1 \text{ (rounds to 1)}$$

### Problem 3

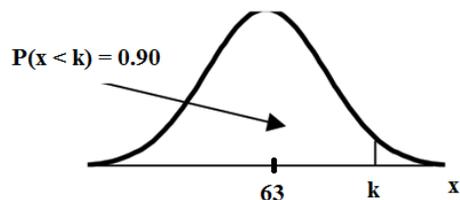
Find the 90th percentile (that is, find the score  $k$  that has 90 % of the scores below  $k$  and 10% of the scores above  $k$ ).

#### Solution

Find the 90th percentile. For each problem or part of a problem, draw a new graph. Draw the x-axis. Shade the area that corresponds to the 90th percentile.

**Let  $k$  = the 90th percentile.**  $k$  is located on the x-axis.  $P(x < k)$  is the area to the left of  $k$ . The 90th percentile  $k$  separates the exam scores into those that are the same or lower than  $k$  and those that are the same or higher. Ninety percent of the test scores are the same or lower than  $k$  and 10% are the same or higher.  $k$  is often called a **critical value**.

$$k = 69.4 \text{ (calculator or computer)}$$



The 90th percentile is 69.4. This means that 90% of the test scores fall at or below 69.4 and 10% fall at or above. For the TI-83+ or TI-84 calculators, use invNorm in 2nd DISTR. invNorm(area to the left, mean, standard deviation) For this problem, invNorm(0.90,63,5) = 69.4

**Problem 4**

Find the 70th percentile (that is, find the score  $k$  such that 70% of scores are below  $k$  and 30% of the scores are above  $k$ ).

**Solution**

Find the 70th percentile.

Draw a new graph and label it appropriately.  $k = 65.6$

The 70th percentile is 65.6. This means that 70% of the test scores fall at or below 65.5 and 30% fall at or above.

$$\text{invNorm}(0.70, 63, 5) = 65.6$$

**Example 3**

A computer is used for office work at home, research, communication, personal finances, education, entertainment, social networking and a myriad of other things. Suppose that the average number of hours a household personal computer is used for entertainment is 2 hours per day. Assume the times for entertainment are normally distributed and the standard deviation for the times is half an hour.

**Problem 1**

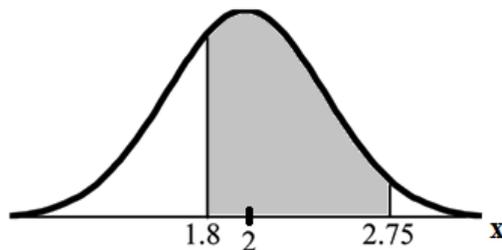
Find the probability that a household personal computer is used between 1.8 and 2.75 hours per day.

**Solution**

Let  $X$  = the amount of time (in hours) a household personal computer is used for entertainment.  $x \sim N(2, 0.5)$  where  $\mu = 2$  and  $\sigma = 0.5$ .

Find  $P(1.8 < x < 2.75)$ .

The probability for which you are looking is the area **between**  $x = 1.8$  and  $x = 2.75$ .  $P(1.8 < x < 2.75) = 0.5886$



$$\text{normalcdf}(1.8, 2.75, 2, 0.5) = 0.5886$$

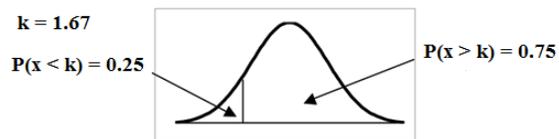
The probability that a household personal computer is used between 1.8 and 2.75 hours per day for entertainment is 0.5886.

**Problem 2**

Find the maximum number of hours per day that the bottom quartile of households use a personal computer for entertainment.

**Solution**

To find the maximum number of hours per day that the bottom quartile of households uses a personal computer for entertainment, **find the 25th percentile**,  $k$ , where  $P(x < k) = 0.25$ .



$$\text{invNorm}(0.25, 2, .5) = 1.66$$

The maximum number of hours per day that the bottom quartile of households uses a personal computer for entertainment is 1.66 hours.