Normal Distribution: Homework*

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Exercise 1 ***(Solution on p. 7.)***

According to a study done by De Anza students, the height for Asian adult males is normally distributed with an average of 66 inches and a standard deviation of 2.5 inches. Suppose one Asian adult male is randomly chosen. Let X =height of the individual.

a. $X\sim$_____(_____,_____)
b. Find the probability that the person is between 65 and 69 inches. Include a sketch of the graph and write a probability statement.
c. Would you expect to meet many Asian adult males over 72 inches? Explain why or why not, and justify your answer numerically.
d. The middle 40% of heights fall between what two values? Sketch the graph and write the probability statement.

Exercise 2

IQ is normally distributed with a mean of 100 and a standard deviation of 15. Suppose one individual is randomly chosen. Let X =IQ of an individual.

a. $X\sim$_____(_____,_____)
b. Find the probability that the person has an IQ greater than 120. Include a sketch of the graph and write a probability statement.
c. Mensa is an organization whose members have the top 2% of all IQs. Find the minimum IQ needed to qualify for the Mensa organization. Sketch the graph and write the probability statement.
d. The middle 50% of IQs fall between what two values? Sketch the graph and write the probability statement.

Exercise 3 ***(Solution on p. 7.)***

The percent of fat calories that a person in America consumes each day is normally distributed with a mean of about 36 and a standard deviation of 10. Suppose that one individual is randomly chosen. Let X =percent of fat calories.

a. $X\sim$_____(_____,_____)
b. Find the probability that the percent of fat calories a person consumes is more than 40. Graph the situation. Shade in the area to be determined.

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c. Find the maximum number for the lower quarter of percent of fat calories. Sketch the graph and write the probability statement.

Exercise 4

Suppose that the distance of fly balls hit to the outfield (in baseball) is normally distributed with a mean of 250 feet and a standard deviation of 50 feet.

a. If X = distance in feet for a fly ball, then $X\sim$_____(_____,_____)

b. If one fly ball is randomly chosen from this distribution, what is the probability that this ball traveled fewer than 220 feet? Sketch the graph. Scale the horizontal axis X. Shade the region corresponding to the probability. Find the probability.

c. Find the 80th percentile of the distribution of fly balls. Sketch the graph and write the probability statement.

Exercise 5 ***(Solution on p. 7.)***

In China, 4-year-olds average 3 hours a day unsupervised. Most of the unsupervised children live in rural areas, considered safe. Suppose that the standard deviation is 1.5 hours and the amount of time spent alone is normally distributed. We randomly survey one Chinese 4-year-old living in a rural area. We are interested in the amount of time the child spends alone per day. (Source: **San Jose Mercury News**)

a. In words, define the random variable X. $X =$

b. $X\sim$

c. Find the probability that the child spends less than 1 hour per day unsupervised. Sketch the graph and write the probability statement.

d. What percent of the children spend over 10 hours per day unsupervised?

e. 70% of the children spend at least how long per day unsupervised?

Exercise 6

In the 1992 presidential election, Alaska's 40 election districts averaged 1956.8 votes per district for President Clinton. The standard deviation was 572.3. (There are only 40 election districts in Alaska.) The distribution of the votes per district for President Clinton was bell-shaped. Let X = number of votes for President Clinton for an election district. (Source: **The World Almanac and Book of Facts**)

a. State the approximate distribution of X. $X\sim$

b. Is 1956.8 a population mean or a sample mean? How do you know?

c. Find the probability that a randomly selected district had fewer than 1600 votes for President Clinton. Sketch the graph and write the probability statement.

d. Find the probability that a randomly selected district had between 1800 and 2000 votes for President Clinton.

e. Find the third quartile for votes for President Clinton.

Exercise 7 ***(Solution on p. 7.)***

Suppose that the duration of a particular type of criminal trial is known to be normally distributed with a mean of 21 days and a standard deviation of 7 days.

a. In words, define the random variable X. $X =$

b. $X\sim$

c. If one of the trials is randomly chosen, find the probability that it lasted at least 24 days. Sketch the graph and write the probability statement.

d. 60% of all of these types of trials are completed within how many days?

Exercise 8

Terri Vogel, an amateur motorcycle racer, averages 129.71 seconds per 2.5 mile lap (in a 7 lap race) with a standard deviation of 2.28 seconds . The distribution of her race times is normally distributed. We are interested in one of her randomly selected laps. (Source: log book of Terri Vogel)

a. In words, define the random variable X. $X =$
b. $X\sim$
c. Find the percent of her laps that are completed in less than 130 seconds.
d. The fastest 3% of her laps are under _____ .
e. The middle 80% of her laps are from _____ seconds to _____ seconds.

Exercise 9 ***(Solution on p. 7.)***

Thuy Dau, Ngoc Bui, Sam Su, and Lan Voung conducted a survey as to how long customers at Lucky claimed to wait in the checkout line until their turn. Let X =time in line. Below are the ordered real data (in minutes):

0.50	4.25	5	6	7.25
1.75	4.25	5.25	6	7.25
2	4.25	5.25	6.25	7.25
2.25	4.25	5.5	6.25	7.75
2.25	4.5	5.5	6.5	8
2.5	4.75	5.5	6.5	8.25
2.75	4.75	5.75	6.5	9.5
3.25	4.75	5.75	6.75	9.5
3.75	5	6	6.75	9.75
3.75	5	6	6.75	10.75

Table 1

a. Calculate the sample mean and the sample standard deviation.
b. Construct a histogram. Start the $x-$ axis at -0.375 and make bar widths of 2 minutes.
c. Draw a smooth curve through the midpoints of the tops of the bars.
d. In words, describe the shape of your histogram and smooth curve.
e. Let the sample mean approximate μ and the sample standard deviation approximate σ. The distribution of X can then be approximated by $X\sim$
f. Use the distribution in (e) to calculate the probability that a person will wait fewer than 6.1 minutes.
g. Determine the cumulative relative frequency for waiting less than 6.1 minutes.
h. Why aren't the answers to (f) and (g) exactly the same?
i. Why are the answers to (f) and (g) as close as they are?
j. If only 10 customers were surveyed instead of 50, do you think the answers to (f) and (g) would have been closer together or farther apart? Explain your conclusion.

Exercise 10

Suppose that Ricardo and Anita attend different colleges. Ricardo's GPA is the same as the average GPA at his school. Anita's GPA is 0.70 standard deviations above her school average. In complete sentences, explain why each of the following statements may be false.

a. Ricardo's actual GPA is lower than Anita's actual GPA.
b. Ricardo is not passing since his z-score is zero.
c. Anita is in the 70th percentile of students at her college.

Exercise 11 ***(Solution on p. 7.)***

Below is a sample of the maximum capacity (maximum number of spectators) of sports stadiums. The table does not include horse racing or motor racing stadiums. (Source: **http://en.wikipedia.org/wiki/List_of_s**

40,000	40,000	45,050	45,500	46,249	48,134
49,133	50,071	50,096	50,466	50,832	51,100
51,500	51,900	52,000	52,132	52,200	52,530
52,692	53,864	54,000	55,000	55,000	55,000
55,000	55,000	55,000	55,082	57,000	58,008
59,680	60,000	60,000	60,492	60,580	62,380
62,872	64,035	65,000	65,050	65,647	66,000
66,161	67,428	68,349	68,976	69,372	70,107
70,585	71,594	72,000	72,922	73,379	74,500
75,025	76,212	78,000	80,000	80,000	82,300

Table 2

a. Calculate the sample mean and the sample standard deviation for the maximum capacity of sports stadiums (the data).
b. Construct a histogram of the data.
c. Draw a smooth curve through the midpoints of the tops of the bars of the histogram.
d. In words, describe the shape of your histogram and smooth curve.
e. Let the sample mean approximate μ and the sample standard deviation approximate σ. The distribution of X can then be approximated by $X\sim$
f. Use the distribution in (e) to calculate the probability that the maximum capacity of sports stadiums is less than 67,000 spectators.
g. Determine the cumulative relative frequency that the maximum capacity of sports stadiums is less than 67,000 spectators. Hint: Order the data and count the sports stadiums that have a maximum capacity less than 67,000. Divide by the total number of sports stadiums in the sample.
h. Why aren't the answers to (f) and (g) exactly the same?

1 Try These Multiple Choice Questions

The questions below refer to the following: The patient recovery time from a particular surgical procedure is normally distributed with a mean of 5.3 days and a standard deviation of 2.1 days.

Exercise 12 ***(Solution on p. 7.)***

What is the median recovery time?

A. 2.7

B. 5.3
C. 7.4
D. 2.1

Exercise 13 *(Solution on p. 7.)*
What is the z-score for a patient who takes 10 days to recover?

A. 1.5
B. 0.2
C. 2.2
D. 7.3

Exercise 14 *(Solution on p. 7.)*
What is the probability of spending more than 2 days in recovery?

A. 0.0580
B. 0.8447
C. 0.0553
D. 0.9420

Exercise 15 *(Solution on p. 7.)*
The 90th percentile for recovery times is?

A. 8.89
B. 7.07
C. 7.99
D. 4.32

The questions below refer to the following: The length of time to find a parking space at 9 A.M. follows a normal distribution with a mean of 5 minutes and a standard deviation of 2 minutes.

Exercise 16 *(Solution on p. 7.)*
Based upon the above information and numerically justified, would you be surprised if it took less than 1 minute to find a parking space?

A. Yes
B. No
C. Unable to determine

Exercise 17 *(Solution on p. 7.)*
Find the probability that it takes at least 8 minutes to find a parking space.

A. 0.0001
B. 0.9270
C. 0.1862
D. 0.0668

Exercise 18 *(Solution on p. 7.)*
Seventy percent of the time, it takes more than how many minutes to find a parking space?

A. 1.24
B. 2.41
C. 3.95

D. 6.05

Exercise 19 ***(Solution on p. 8.)***

If the mean is significantly greater than the standard deviation, which of the following statements is true?

I . The data cannot follow the uniform distribution.
II . The data cannot follow the exponential distribution..
III . The data cannot follow the normal distribution.

A. I only
B. II only
C. III only
D. I, II, and III

Solutions to Exercises in this Module

Solution to Exercise (p. 1)

a. $N(66, 2.5)$
b. 0.5404
c. No
d. Between 64.7 and 67.3 inches

Solution to Exercise (p. 1)

a. N (36,10)
b. 0.3446
c. 29.3

Solution to Exercise (p. 2)

a. the time (in hours) a 4-year-old in China spends unsupervised per day
b. $N(3, 1.5)$
c. 0.0912
d. 0
e. 2.21 hours

Solution to Exercise (p. 2)

a. The duration of a criminal trial
b. $N(21, 7)$
c. 0.3341
d. 22.77

Solution to Exercise (p. 3)

a. The sample mean is 5.51 and the sample standard deviation is 2.15
e. $N(5.51, 2.15)$
f. 0.6081
g. 0.64

Solution to Exercise (p. 4)

a. The sample mean is 60,136.4 and the sample standard deviation is 10,468.1.
e. $N(60136.4, 10468.1)$
f. 0.7440
g. 0.7167

Solution to Exercise (p. 4)

B

Solution to Exercise (p. 5)

C

Solution to Exercise (p. 5)

D

Solution to Exercise (p. 5)

C

Solution to Exercise (p. 5)

A

Solution to Exercise (p. 5)

D

Solution to Exercise (p. 5)
C
Solution to Exercise (p. 6)
B