Hypothesis Testing: Two Population Means and Two Population Proportions: Comparing Two Independent Population Proportions*

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1. The two independent samples are simple random samples that are independent.
2. The number of successes is at least five and the number of failures is at least five for each of the samples.

Comparing two proportions, like comparing two means, is common. If two estimated proportions are different, it may be due to a difference in the populations or it may be due to chance. A hypothesis test can help determine if a difference in the estimated proportions $(P_A - P_B)$ reflects a difference in the population proportions.

The difference of two proportions follows an approximate normal distribution. Generally, the null hypothesis states that the two proportions are the same. That is, $H_o : p_A = p_B$. To conduct the test, we use a pooled proportion, p_c.

The pooled proportion is calculated as follows:

$$p_c = \frac{x_A + x_B}{n_A + n_B} \tag{1}$$

The distribution for the differences is:

$$P'_A - P'_B \sim N\left[0, \sqrt{p_c \cdot (1 - p_c) \cdot \left(\frac{1}{n_A} + \frac{1}{n_B}\right)}\right] \tag{2}$$

*Version 1.12: Jun 14, 2012 8:55 pm -0500
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The test statistic (z-score) is:

$$z = \frac{(p'_A - p'_B) - (p_A - p_B)}{\sqrt{p_c \cdot (1 - p_c) \cdot \left(\frac{1}{n_A} + \frac{1}{n_B}\right)}} \tag{3}$$

Example 1: Two population proportions
Two types of medication for hives are being tested to determine if there is a **difference in the proportions of adult patient reactions. Twenty** out of a random **sample of 200** adults given medication A still had hives 30 minutes after taking the medication. **Twelve** out of another **random sample of 200 adults** given medication B still had hives 30 minutes after taking the medication. Test at a 1% level of significance.

1 Determining the solution

This is a test of 2 population proportions.

Problem ***(Solution on p. 4.)***

How do you know?

Let A and B be the subscripts for medication A and medication B. Then p_A and p_B are the desired population proportions.

Random Variable:

$P'_A - P'_B$ = difference in the proportions of adult patients who did not react after 30 minutes to medication A and medication B.

$H_o : p_A = p_B$ $\quad$ $p_A - p_B = 0$

$H_a : p_A \neq p_B$ $\quad$ $p_A - p_B \neq 0$

The words **"is a difference"** tell you the test is two-tailed.

Distribution for the test: Since this is a test of two binomial population proportions, the distribution is normal:

$p_c = \frac{x_A + x_B}{n_A + n_B} = \frac{20+12}{200+200} = 0.08 \quad 1 - p_c = 0.92$

Therefore, $\quad P'_A - P'_B \sim N\left[0, \sqrt{(0.08) \cdot (0.92) \cdot \left(\frac{1}{200} + \frac{1}{200}\right)}\right]$

$P'_A - P'_B$ follows an approximate normal distribution.

Calculate the p-value using the normal distribution: p-value = 0.1404.

Estimated proportion for group A: $\quad p'_A = \frac{x_A}{n_A} = \frac{20}{200} = 0.1$

Estimated proportion for group B: $\quad p'_B = \frac{x_B}{n_B} = \frac{12}{200} = 0.06$

Graph:

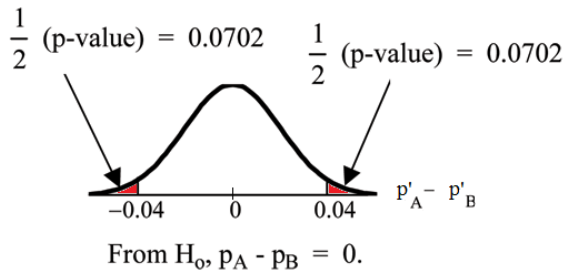

Figure 1

P'$_A$ − P'$_B$ = 0.1 − 0.06 = 0.04.

Half the p-value is below -0.04 and half is above 0.04.

Compare α and the p-value: $\alpha = 0.01$ and the p-value = 0.1404. $\alpha <$ p-value.

Make a decision: Since $\alpha <$ p-value, do not reject H_o.

Conclusion: At a 1% level of significance, from the sample data, there is not sufficient evidence to conclude that there is a difference in the proportions of adult patients who did not react after 30 minutes to medication A and medication B.

NOTE: TI-83+ and TI-84: Press `STAT`. Arrow over to `TESTS` and press `6:2-PropZTest`. Arrow down and enter `20` for x1, `200` for n1, `12` for x2, and `200` for n2. Arrow down to `p1`: and arrow to `not equal p2`. Press `ENTER`. Arrow down to `Calculate` and press `ENTER`. The p-value is $p = 0.1404$ and the test statistic is 1.47. Do the procedure again but instead of `Calculate` do `Draw`.

Solutions to Exercises in this Module

Solution to Example 1, Problem (p. 2)

The problem asks for a difference in proportions.