Connexions module: m17305

POWER DISSIPATION IN RESISTOR CIRCUITS*

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Abstract

Power dissipation in resistor circuits.

We can find voltages and currents in simple circuits containing resistors and voltage or current sources. We should examine whether these circuits variables obey the Conservation of Power principle: since a circuit is a closed system, it should not dissipate or create energy. For the moment, our approach is to investigate first a resistor circuit's **power** consumption/creation. Later, we will **prove** that because of **KVL** and **KCLall** circuits conserve power.

As defined on here¹, the instantaneous power consumed/created by every circuit element equals the product of its voltage and current. The total power consumed/created by a circuit equals the sum of each element's power.

$$P = \sum_{k} v_k i_k$$

Recall that each element's current and voltage must obey the convention that positive current is defined to enter the positive-voltage terminal. With this convention, a positive value of $v_k i_k$ corresponds to consumed power, a negative value to created power. Because the total power in a circuit must be zero (P=0), some circuit elements must create power while others consume it.

Consider the simple series circuit should in here². In performing our calculations, we defined the current i_{out} to flow through the positive-voltage terminals of both resistors and found it to equal $i_{\text{out}} = \frac{v_{\text{in}}}{R_1 + R_2}$. The voltage across the resistor R_2 is the output voltage and we found it to equal $v_{\text{out}} = \frac{R_2}{R_1 + R_2}v_{\text{in}}$. Consequently, calculating the power for this resistor yields

$$P_2 = \frac{R_2}{(R_1 + R_2)^2} v_{\rm in}^2$$

Consequently, this resistor dissipates power because P_2 is positive. This result should not be surprising since we showed³ that the power consumed by **any** resistor equals either of the following.

$$\frac{v^2}{R} \text{ or } i^2 R \tag{1}$$

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^{1&}quot;Voltage, Current, and Generic Circuit Elements" http://cnx.org/content/m0011/latest/#para5

^{2&}quot;Electric Circuits and Interconnection Laws", Figure 1 http://cnx.org/content/m0014/latest/#simplecircuit

^{3&}quot;Ideal Circuit Elements" http://cnx.org/content/m0012/latest/#para2a

Connexions module: m17305 2

Since resistors are positive-valued, **resistors always dissipate power**. But where does a resistor's power go? By Conservation of Power, the dissipated power must be absorbed somewhere. The answer is not directly predicted by circuit theory, but is by physics. Current flowing through a resistor makes it hot; its power is dissipated by heat.

NOTE: A physical wire has a resistance and hence dissipates power (it gets warm just like a resistor in a circuit). In fact, the resistance of a wire of length L and cross-sectional area A is given by

$$R = \frac{\rho L}{A}$$

The quantity ρ is known as the **resistivity** and presents the resistance of a unit-length, unit cross-sectional area material constituting the wire. Resistivity has units of ohm-meters. Most materials have a positive value for ρ , which means the longer the wire, the greater the resistance and thus the power dissipated. The thicker the wire, the smaller the resistance. Superconductors have zero resistivity and hence do not dissipate power. If a room-temperature superconductor could be found, electric power could be sent through power lines without loss!

Exercise 1 (Solution on p. 3.)

Calculate the power consumed/created by the resistor R_1 in our simple circuit example.

We conclude that both resistors in our example circuit consume power, which points to the voltage source as the producer of power. The current flowing **into** the source's positive terminal is $-i_{\text{out}}$. Consequently, the power calculation for the source yields

$$-(v_{\rm in}i_{
m out}) = -\left(\frac{1}{R_1 + R_2}v_{
m in}^2\right)$$

We conclude that the source provides the power consumed by the resistors, no more, no less.

Exercise 2 (Solution on p. 3.)

Confirm that the source produces exactly the total power consumed by both resistors.

This result is quite general: sources produce power and the circuit elements, especially resistors, consume it. But where do sources get their power? Again, circuit theory does not model how sources are constructed, but the theory decrees that **all** sources must be provided energy to work.

Connexions module: m17305

Solutions to Exercises in this Module

Solution to Exercise (p. 2)

The power consumed by the resistor R_1 can be expressed as

$$(v_{\rm in} - v_{\rm out}) i_{\rm out} = \frac{R_1}{(R_1 + R_2)^2} v_{\rm in}^2$$

Solution to Exercise (p. 2)

$$\frac{1}{R_1 + R_2} v_{\text{in}}^2 = \frac{R_1}{(R_1 + R_2)^2} v_{\text{in}}^2 + \frac{R_2}{(R_1 + R_2)^2} v_{\text{in}}^2$$