

DYNAMICS*

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Abstract

This module presents the basic idea of dynamics: the time evolution of a phenomenon. Part of the notes used in a course taught at Rice University in 1973-74.

1 Dynamics of Systems

In this module we will present several definitions and a language that will later be used to model social systems. Although a complete and detailed presentation will not be made, the ideas covered are very important for anything other than a superficial understanding of dynamic models. Much of this material grew out of what is called system theory and control theory. [1][2]

1.1 Definitions

As we noted before, it is sometimes difficult to give clear, precise definitions of some ideas. That is the case for the definition of a system which sounds a bit vague but seems to be as good as possible.

A system is defined as a set of interrelated entities, variables, or ideas that have some common features or purpose.

Examples of systems would be a car, a radio, a transportation network, a set of coupled equations, a society, a family, etc. The system may be physical, biological, social, conceptual, or many other forms.

The dynamics of a system is the way the various variables of the system change and evolve with time.

The study of dynamics is an important part of physics, engineering, and economics. Indeed, the study of change in history, psychology, etc. can be viewed as a study of dynamics, and when anyone makes predictions about the future, he is certainly using a dynamic model whether he realizes it or not. There are many studies of systems which are not dynamic models - these use static or equilibrium models and study relationships where time variations are assumed not important. The mathematics often used in the study of system dynamics are calculus and differential equations.

The structure of a system is the specification of the components of importance and interest and the description of the relations and interconnections within the system.

The choice of structures may be easy or very difficult depending on the system. In many physical systems the structure is fairly well developed, however, for social systems it is more complicated. The choice of age groupings, economic groupings, etc. by a sociologist is the choice of structure for a particular system. Indeed, much of the research in the social sciences has centered around structure with relatively little work being done explicitly on dynamics. For our purposes, we need both.

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1.2 Descriptions of Systems

There are two rather different but complimentary descriptions that have been used with success in systems analysis. One is an input-output or external approach, and the other is a state variable or interval approach. Both have merits and will be briefly described.

1.2.1 The Input-Output Description

Here there are three entities considered: the input x , output y , and the system s . Symbolically, this is illustrated by

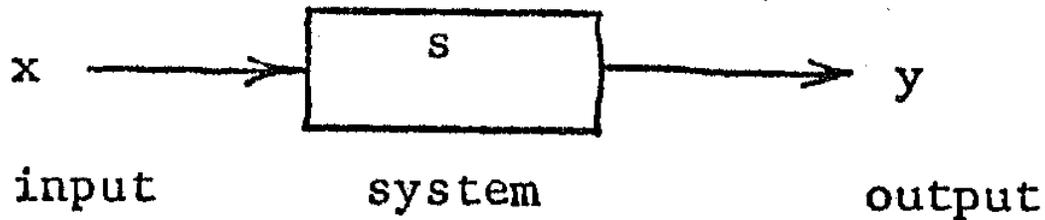


Figure 1

This has proven a very valuable approach that avoids internal details of the system that are of no interest or are difficult to describe.

There are three problems that can be formulated with this description:

1. Analysis: x and s given, find y ;
2. Synthesis: x and y given, find s ; and
3. Control: s and y given, find x .

In the modeling of systems and signals, one often has a partial description of all three, and they must be completed in a way to be consistent.

1.2.2 The State Variable or Internal

In this case, a detailed description of the internal structure of a model is made. The idea of a "state" is very important to dynamic systems, but is so fundamental as to be difficult to define. The situation is further complicated by the fact that the word state is used in many different ways in other areas.

The state of system is the present information about the past that allows one to predict the effect of the past on the future. The variables that describe the state are called the state variables and the minimum number of state variables is called the order (or dimension) of the system.

For example, if one is modeling a social system, in order to predict the future population, in addition to other factors, one must know the present population; therefore, population would be a state variable. Another example might be a second-order differential equation.

$$x'' + ax' + bx = 0$$

Here $x(0)$ and $\dot{x}(0)$ are needed to calculate $x(t)$; therefore, they could be state variables. A mechanical example would be a moving mass where one would have to know the position x and velocity v at some time to predict its future position.

In addition to state variables, a system often has many variables that are derived from present values of other variables, but do not require any past values. These are very important in the description of some systems, and it is often very difficult to distinguish between state and derived variables when initially trying to set up a model for a complex system.

The difficulty in choosing state variables is further compounded by the fact that they are not unique. (Their number is, however.) For example, in a system of equations, a change of variables could be carried out and the new variables used as states. In the mechanical example, one could choose $v + x$ for one state variable and $\frac{1}{2}v - \sqrt{2x}$ for the other, although it's hard to imagine why one would want to.

1.3 Deterministic and Probabilistic

Still another division of description is into those that use deterministic equations to relate the various system variables and those that relate the statistics of the variables. These two approaches are complimentary. For example, in describing a gas in a container, one can relate the gross characteristics of pressure, temperature and volume by an algebraic equation; however, one must resort to statistics to describe an individual molecule. In the case of the social model, it seems to also hold that individual people or small groups must be described statistically, but the gross behavior of large aggregates can be described deterministically. This is certainly not as clear-cut as for a container of gas, but it is what we will follow.

Indeed, not only is the decision between a deterministic and probabilistic model difficult to make for a social system, but the choice of structure, state variables, and many other factors are all difficult and the subject of much debate a long researchers. What this means, however, is the basic concepts and definitions must be understood even better and used with even greater care.

1.4 Classifications

There are a number of rather common classifications of systems that prove useful. The two most important are given here in terms of an input-output description.

- A.** A system is called linear if, and only if, the following two conditions hold. In an input x_1 causes an output y_1 , and an input x_2 causes an output y_2 , then an input which is the sum of two inputs, $x_1 + x_2$, must cause an output $y_1 + y_2$. This is called superposition. If the input x_1 is scaled by an arbitrary value a , then the resulting output must also be scaled by the same value a .

$$F = M \cdot A \quad (1)$$

$$\text{If } x_1 \rightarrow y_1 \quad \text{and} \quad x_2 \rightarrow y_2 \quad (2)$$

$$\text{then } (x_1 + x_2) \rightarrow (y_1 + y_2) \quad (3)$$

$$\text{and } ax_1 \rightarrow ay_1 \quad (4)$$

- B.** A system is called time-invariant or stationary if, and only if, the following is true for arbitrary t .

$$\text{If } x(t) \rightarrow y(t) \quad \text{then} \quad x(t+T) \rightarrow y(t+T) .$$

1.5 Feedback

A particular structure [Luenberger 1979] which is so important that it warrants special discussion has the feature that the output affects the input. This is illustrated by the following figure.

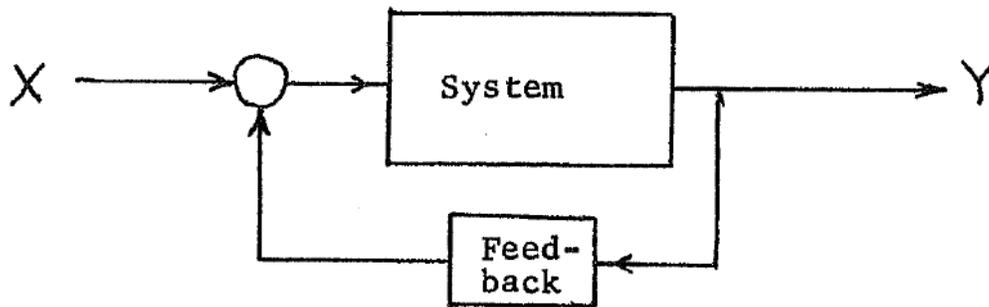


Figure 2

Feedback is often part of naturally occurring systems and it also is often a part of constructed systems. The most common feedback system is probably the thermostat that uses a measured temperature to feedback a controlling signal to a heater in an oven or room heating system. The filling mechanism in the tank of a toilet uses a float to feedback a measure of the water level to control the input valve. A person's blood sugar level is controlled by a complicated biological feedback system. The power steering of a car, the auto-pilot of an airplane, and the control of a satellite rocket are all examples of feedback.

An interesting model using feedback can be used to describe a bank savings account. Here the output can be the amount of interest earned which is then fed back and added to increase the account. This feedback is called compounding, and results in the rapid exponential growth of an account.

A similar model will be used to describe a population where the feedback signal is the number of people added by births less the number of deaths. This forms the basis of the exponential predictions of population growth, and we will explore it in detail later.

The basis of the free marketplace is based on feedback through price changes to cause the supply to follow the demand.

While feedback is a useful concept, its effects become more difficult to predict as the systems become more complex. A simple example illustrates one problem. Consider a person adjusting the temperature of his shower by the hot and cold valves. If there is a time delay introduced by a length of pipe between the valves and the shower head, the person will over control. If the water is initially too hot, he will turn on the cold water, but because of the delay, no effect is immediately felt so more cold water is turned on. This continues until finally the now very cold water reaches the shower head, whereupon the person starts the same procedure of increasing the hot water. This oscillation will continue until the person "gets smart" and allows for the delay. A similar problem can occur in college education because of the four-year delay between the choice of a major and the graduation to a job.

A bit of reflection begins to show the complicated nature of a social system will involve multi-variable nonlinear systems with time delays and multiple feedback loops.

References

- [1] Director and Roher. *Introduction to System Theory*. McGraw-Hill, 1972.
- [2] Tokad Koenig and Keswan. *Analysis of Discrete Physical Systems*. McGraw-Hill, 1967.