Simultaneous Equations Concepts – Simultaneous Equations by Graphing*

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Abstract

This module introduces how to graph simultaneous equations.

Consider the equation $y = 2\sqrt{x}$. How many (x, y) pairs are there that satisfy this equation? Answer: $(0, 0)$, $(1, 2)$, $(4, 4)$, and $(9, 6)$ are all solutions; and there is an **infinite number of other solutions**. (And don't forget non-integer solutions, such as $(, 1)$!)

Now, consider the equation $y = x + \frac{1}{2}$. How many pairs satisfy **this** equation? Once again, an infinite number. Most equations that relate two variables have an infinite number of solutions.

To consider these two equations "simultaneously" is to ask the question: what (x, y) pairs make **both equations true**? To express the same question in terms of functions: what values can you hand the functions $2\sqrt{x}$ and $x + \frac{1}{2}$ that will make these two functions produce the **same answer**?

*Version 1.2: Dec 30, 2008 5:05 pm -0600

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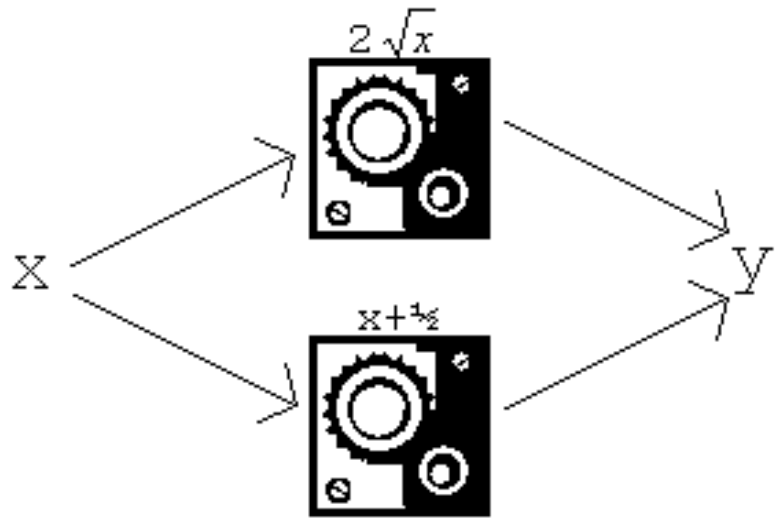

Figure 1: What number goes into **both functions** and makes them give the **same answer**? Is there even such a number? Is there more than one such number?

At first glance, it is not obvious how to approach such a question– it is not even obvious how many answers there will be.

One way to answer such a question is by graphing. Remember, the graph of $y = 2\sqrt{x}$ is the set of all points that satisfy that relationship; and the graph of $y = x + \frac{1}{2}$ is the set of all points that satisfy that relationship. So **the intersection(s) of these two graphs is the set of all points that satisfy both relationships**.

How can we graph these two? The second one is easy: it is a line, already in $y = mx + b$ format. The y-intercept is $\frac{1}{2}$ and the slope is 1. We can graph the first equation by plotting points; or, if you happen to know what the graph of $y = \sqrt{x}$ looks like, you can stretch the graph vertically to get $y = 2\sqrt{x}$, since all the y-values will double. Either way, you wind up with something like this:

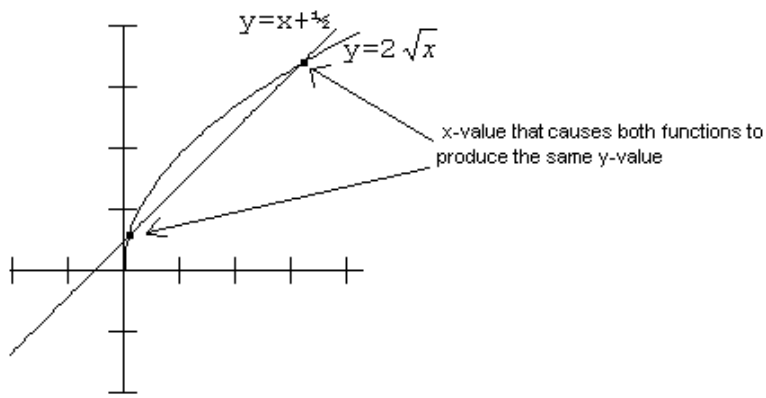

Figure 2

We can see that there are two points of intersection. One occurs when x is barely greater than 0 (say, $x = 0.1$), and the other occurs at approximately $x = 3$. There will be no more points of intersection after this, because the line will rise faster than the curve.

Exercise 1 ***(Solution on p. 4.)***

$y = 2\sqrt{x}$
$y = x + \frac{1}{2}$

Graphing has three distinct advantages as a method for solving simultaneous equations.

1. It works on any type of equations.
2. It tells you **how many** solutions there are, as well as what the solutions are.
3. It can help give you an **intuitive feel** for why the solutions came out the way they did.

However, graphing also has two **dis**advantages.

1. It is time-consuming.
2. It often yields solutions that are **approximate**, not exact—because you find the solutions by simply "eyeballing" the graph to see where the two curves meet.

For instance, if you plug the number 3 into both of these functions, will you get the same answer?
$3 \to 2\sqrt{x} \to 2\sqrt{3} \approx 3.46$
$3 \to x + \frac{1}{2} \to 3.5$

Pretty close! Similarly, $2\sqrt{.1} \approx 0.632$, which is quite close to 0.6. But if we want more exact answers, we will need to draw a much more exact graph, which becomes **very** time-consuming. (Rounded to three decimal places, the actual answers are 0.086 and 2.914.)

For more exact answers, we use analytic methods. Two such methods will be discussed in this chapter: **substitution** and **elimination**. A third method will be discussed in the section on Matrices.

Solutions to Exercises in this Module

Solution to Exercise (p. 3)

From graphing...

$x = 0.1$, $x = 3$