Simultaneous Equations Concepts – Using Letters as Numbers*

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Abstract

This module shows how it can be helpful, on occasion, to use letters as numbers in order to quickly find solutions to variations on a simultaneous equation problem.

Toward the end of this chapter, there are some problems in substitution and elimination where **letters are used in place of numbers**. For instance, consider the following problem:

$$2y - \text{ax} = 7 \tag{1}$$

$$4y + 3\text{ax} = 9 \tag{2}$$

What do we do with those "a"s? Like any other variable, they simply represent an unknown number. As we solve for x, we will simply leave a as a variable.

This problem lends itself more naturally to elimination than to substitution, so I will double the top equation and then subtract the two equations and solve.

$$\begin{array}{c} 4y - 2\text{ax} = 14 \\ \underline{-\left(4y + 3\text{ax} = 9\right)} \\ 0y - 5\text{ax} = 5 \end{array} \tag{3}$$

$$x = \frac{5}{-5a} = \frac{-1}{a} \tag{4}$$

As always, we can solve for the second variable by plugging into either of our original equations.

$$2y - a\left(\frac{-1}{a}\right) = 7 \tag{5}$$

$$2y + 1 = 7 \tag{6}$$

$$y = 3 \tag{7}$$

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There is no new math here, just elimination. The real trick is not to be spooked by the a, and do the math just like you did before.

And what does that **mean?** It means we have found a solution that works for those two equations, regardless of a. We can now solve the following three problems (and an infinite number of others) without going through the hard work.

If $a = 5$**,**	**If** $a = 10$**,**	**If** $a = -3$**,**
The original equations become:	The original equations become:	The original equations become:
$2y - 5x = 7$ $4y + 15x = 9$	$2y - 10x = 7$ $4y + 30x = 9$	$2y + 3x = 7$ $4y - 9x = 9$
And the solution is:	And the solution is:	And the solution is:
$x = \frac{-1}{5}, y = 3$	$x = \frac{-1}{10}, y = 3$	$x = \frac{1}{3}, y = 3$

Table 1

The whole point is that I did not have to **solve** those three problems—by elimination, substitution, or anything else. All I had to do was plug a into the general answer I had already found previously. If I had to solve a hundred such problems, I would have saved myself a great deal of time by going through the hard work once to find a general solution!

Mathematicians use this trick all the time. If they are faced with many similar problems, they will attempt to find a **general** problem that encompasses all the **specific** problems, by using variables to replace the numbers that change. You will do this in an even more general way in the text, when you solve the "general" simultaneous equations where **all** the numbers are variables. Then you will have a formula that you can plug any pair of simultaneous equations into to find the answer at once. This formula would also make it very easy, for instance, to program a computer to solve simultaneous equations (computers are terrible at figuring things out, but they're great at formulas).