

QUADRATIC CONCEPTS – DIFFERENT TYPES OF SOLUTIONS TO QUADRATIC EQUATIONS*

Kenny M. Felder

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Abstract

This module discusses the different types of solutions to quadratic equations.

The heart of the quadratic formula is the part under the square root: $b^2 - 4ac$. This part is so important that it is given its own name: the **discriminant**. It is called this because it **discriminates** the different types of solutions that a quadratic equation can have.

Common Error

Students often think that the discriminant is $\sqrt{b^2 - 4ac}$. But the discriminant is **not the square root**, it is the part that is **under** the square root:

$$\text{Discriminant} = b^2 - 4ac \quad (1)$$

It can often be computed quickly and easily without a calculator.

Why is this quantity so important? Consider the above example, where the discriminant was 36. This means that we wound up with ± 6 in the numerator. So the problem had two different, rational answers: $2\frac{2}{3}$ and $3\frac{1}{3}$.

Now, consider $x^2 + 3x + 1 = 0$. In this case, the discriminant is $3^2 - 4(1)(1) = 5$. We will end up with $\pm\sqrt{5}$ in the numerator. There will still be two answers, but they will be **irrational**—they will be impossible to express as a fraction without a square root.

$4x^2 - 20x + 25 = 0$. Now the discriminant is $20^2 - 4(4)(25) = 400 - 400 = 0$. We will end up with ± 0 in the numerator. But it makes no difference if you add or subtract 0; you get the same answer. So this problem will have only one answer.

And finally, $3x^2 + 5x + 4 = 0$. Now, $b^2 - 4ac = 5^2 - 4(3)(4) = 25 - 48 = -23$. So in the numerator we will have $\sqrt{-23}$. Since you cannot take the square root of a negative number, there will be no solutions!

Summary: The Discriminant

- If the discriminant is a perfect square, you will have two rational solutions.
- If the discriminant is a positive number that is not a perfect square, you will have two irrational solutions (ie they will have square roots in them).
- If the discriminant is 0, you will have one solution.

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- If the discriminant is negative, you will have no solutions.

These rules do not have to be memorized: you can see them very quickly by understanding the quadratic formula (which **does** have to be memorized—if all else fails, try singing it to the tune of Frère Jacques).

Why is it that quadratic equations can have 2 solutions, 1 solution, or no solutions? This is easy to understand by looking at the following graphs. Remember that in each case the quadratic equation asks when the function is 0—that is to say, when it crosses the x -axis.

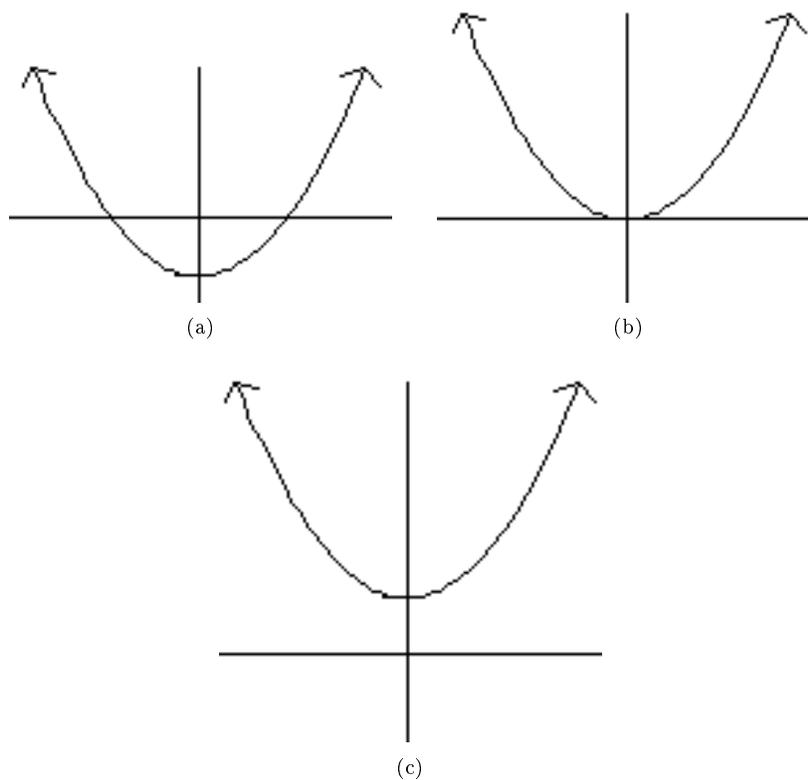


Figure 1: (a) $y = x^2 - 2$ Equals 0 two times (b) $y = x^2$ Equals 0 once (c) $y = x^2 + 2$. Never equals 0

More on how to generate these graphs is given below. For the moment, the point is that you can visually see why a quadratic function can equal 0 twice, or one time, or never. It can **not** equal 0 three or more times.